LIMIT DESIGN OF SHIP TRANSVERSE WEB FRAMES

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ABSTRACT

An analytical investigation is undertaken into the application of the upper bound theorem of limit design for use in the design of ship transverse web frame structures. The revised simplex method is used to solve the upper bound theorem relationships for required values of plastic limit moment for structurally safe designs. Bracket and beam overlap influence is idealized by assuming rigid extensions in the overlap and bracket regions. Comparisons are made with existing web frames designed by elastic methods.

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NOTATION

A	Total beam cross sectional area.
A_F	Beam Flange Area.
A_T	Cross sectional area of beam web and top flanges.
$\underline{A}$	Coefficient matrix of constraint equation.
a	Distance between centroids.
a_{ij}	Constants of constraint function.
$\underline{B}$	Vector of the basis.
$\underline{b}$	Right hand side of constraint relation.
b_i	Constant of constraint function.
c_i, c_j, c_k	Length of beam overlap.
$\underline{c}$	Coefficient matrix of objective function.
d_1	Bracket leg length.
f	Objective function.
f_I, f_{II}	Lengths of rigid sections due to beam overlap.
g	Beam element weight per unit length.
h	Beam depth.
L_i, l_i	Beam element length.
M_a, M_b, M_i, M_0	Plastic limit moment.
$\underline{P}_j$	Vector associated with extreme point.
s_I, s_{II}	Overlapping web depths.
S2	Cross-tie height above base line.
S5	Side tank width.

NOTATION (cont.)

t_1^b	Rigid extension due to bracket influence.
u	Normal velocity.
w	In line velocity.
x_j	Variable of linear programming problem.
z_j	Iteration solution value of objective function.
z_p	Plastic section modulus.
$\underline{\pi}$	Multiplier vector.
ρ	Material density.
σ_y	Material yield stress.

1. INTRODUCTION

1.1 Preface

The design problem of transverse web frame structures in ships is receiving an increasing amount of interest with the advent of mammoth tankers and bulk carriers. Transverse framing accounts for a sizable portion of the total structural weight of such ships. A substantial weight savings can result from the proper design and optimization of the repeating transverse frames in the prismatic mid section of these large vessels. Great effort is usually made in the design phase to make these structures as efficient as possible. Modern day design efforts have been concentrated in two methods of elastic design.

The finite element method is the most popular of the modern elastic ship design methods. A number of review papers [1 - 4] have discussed the application of this technique to the design of ship structures in general. Smith & Woodhead [1] discussed specific application of the method to web frames.

The second elastic design method uses beam theory and was first discussed in depth by Lund [5]. Others have also written on the topic [4 - 14].

Both of the elastic design methods involve systems of nonlinear equations. Design optimization can

become complex and expensive in terms of computation time. The nonlinearities require the use of optimization techniques such as nonlinear programming, Powell's direct search method or the gradient method.

The elastic design approach and the nonlinear optimization methods combine to form a laborous design task with a trusted but possibly expensive and inefficient product.

The purpose of this study is to examine the use of another design method, namely plastic limit design, in the design of transverse web frames in ships.

1.2 Historical Review

A search of existing literature reveals a wealth of information on the use of plasticity design methods and specifically the use of the plastic limit design theorems dating back several decades [15 - 22]. The limit design methods are common place and vital in nearly all sectors of structural design except naval architecture and ocean engineering. In papers on marine structures where the use of limit design is suggested [23 - 25], it is often from the civil engineering point of view with examples taken from standard civil engineering texts.

Jones [24, 25] uses the limit theorems in the analysis of tanker web frames in two separate papers. To date, his are the only papers known to discuss in detail the use of the limit theorems on marine structures loaded

by various conditions of hydrostatic loading.

2. METHOD OF CALCULATION

2.1 Limit Design Upper Bound Theorem

In structures composed of an elastic, perfectly plastic material, one of the primary functions of the structure is that it should carry a given set of loads while in equilibrium without flow occurring in the structure. The structural design problem is referred to as the limit design problem if this is the only function of the structure [22]. An alternative formulation of the limit design problem states that if under all possible failure modes the internal energy dissipated during kinematically admissible, small virtual displacements exceeds the external work rate associated with these displacements then the structure is safe [7]. This latter statement can be used to formulate an upper bound on the system plastic moments as a programming problem.

The limit design upper bound theorem is the primary tool to this analysis. The desired output of the limit design problem is the required weight (i.e. cost) of individual structural elements and the total system.

The output of the limit design process is the plastic moment, therefore, it is necessary to establish some relationship between weight per unit length g and plastic moment M_0. Horne [21] states that for materials of given yield stress and geometrically similar section,

$g \propto M^{2/3}$. For Universal Beam Sections the relationship has been shown to closely approximate $g \propto M^{0.6}$. More generally $g \propto M^n$. Horne also states that the relative plastic moments for absolute minimum weight in a given structure are only very slightly affected by the exact value chosen for the index "n". It will be shown that solutions are greatly simplified by maintaining linear relationships between variables. It, therefore, is assumed that the weight per unit length of an element is given by

$$g = A\,L + B\,L\,M_i$$

where A and B are constants and L is the element length. The total system weight of the structure is then given by

$$G = \sum A_i\,L_i + \sum B_i\,L_i\,M_i$$

The methods of plastic limit design cannot utilize the principle of superposition of stresses and deflections due to individual loads, as is possible in design where the structure remains in the elastic range [15]. A paradox of the limit theorem, however, is that when a collapse mode of a structure involves only part of the structure, the collapse load of the plastic limit moment can be expressed independently of the characteristics of the remainder of the structure [18]. In other words, in formulating the limit design problem, only those loads associated with a given collapse mechanism need be

considered with that mechanism. This premise allows the energy equations associated with each mechanism to be constructed independently.

Further assumptions and idealizations are required to complete the formulation of the limit design problem. Material behavior is idealized to be elastic, perfectly plastic (i.e. free of strain hardening). Strains are assumed to be small. Jones has discussed the importance of these assumptions [26].

Finally, it is assumed that all frames are plane frameworks of beams, in which transverse shear and axial forces do not influence the plastic yielding of the beam. Symonds and Neal [15] stated that shear forces can be neglected except in cases where the ratio of span length to beam depth is less than approximately four to one. Also, the effect of an axial force on the fully plastic moment can be neglected when the force is less than about one fourth of the compressive yield force of the member.

To illustrate the general proceedures of formulating a limit design problem, the simple civil engineering portal frame of Figure 1, will be examined. Application to the more sophisticated ship frames with hydrostatic loads is discusses in Reference [25] in the appendices and in sections 2.2 to 2.4. The frame is restrained vertically as well as horizontally at its

base and the supports are clamped against rotation. The frame is externally determinate and internally indeterminate with $r = 3$ redundancies. A discussion of redundancies may be found in [27].

The frame has $n = 8$ critical sections, or locations where plastic hinges can form. Plastic hinges may form at the ends of a span, directly under a concentrated load or within the span of a member subjected to a distributed load.

Only r of the n critical bending moments can be defined independently of the equilibrium equations [24]. Therefore, there exist $n - r$ or 5 independent relations between the bending moments. These relationships are the system equilibrium equations and can be obtained by utilizing the principle of virtual work. Each equilibrium equation is associated with a kinematically admissible fundamental mechanism. Fundamental or elementary mechanisms are classified by Symonds and Neal as beam, panel, sway and joint rotation. Joint rotations are also classified fictitious. The 5 mechanisms of the sample problem are illustrated in Figure 2. For all 5 failure mechanisms to safely carry the prescribed loads we must have

$$M_4 - 2M_5 + M_6 \geq 2 \qquad (2.1)$$

$$M_1 - 4M_2 + 3M_3 \geq 6 \qquad (2.2)$$

$$M_1 - M_3 + M_7 - M_8 \geq 6 \qquad (2.3)$$

$$M_3 - M_4 \geq 0 \qquad (2.4)$$

$$M_6 - M_7 \geq 0 \qquad (2.5)$$

where values of $\dot{\theta}$, angular velocity have been divided from both sides of the inequalities. Minus signs denote hinge rotations in which the internal fibers of the hinges are in tension. This convention is merely a method of keeping track of the direction of rotation and is not intended to imply negative internal energy dissipation.

The fundamental sway mechanism was eliminated from consideration in this study. This was not because of any assumptions that sway would not cause collapse in ship web frames, but rather that sway could not be induced by any of the symmetrical loading models used by the elastic designs which are used as comparison baselines for this study.

The fundamental mechanisms are not the only mechanisms which must be examined. It is possible for linear combinations of the fundamental mechanisms to control the design [28]. This is true where the combination of mechanisms allows a reduction or elimination of certain plastic hinges reducing the total internal energy dissipation while the combined external work rate remains unchanged. It is here that the signs of the various terms in the equilibrium equations become helpful bookkeeping aids. In combining the fundamental

mechanisms of the example the fictitious joint rotations are used to derive

$$M_3 - 2M_5 + M_7 \geq 2 \qquad (2.6)$$

$$M_1 - 4M_2 + 3M_4 \geq 6 \qquad (2.7)$$

$$M_1 - M_4 + M_7 - M_8 \geq 6 \qquad (2.8)$$

also equations (2.2), (2.3) and (2.5) can be combined to give

$$4M_1 - 4M_2 + 3M_7 - 3M_8 \geq 24 \qquad (2.9)$$

or

$$4M_1 - 4M_2 + 3M_6 - 3M_8 \geq 24 \qquad (2.10)$$

Assume, now, that the structure will be developed with two separate beam sizes. The two vertical elements will be identical in cross section with plastic moment M_a and the horizontal element is allowed to differ from the verticals with plastic moment M_b. Now equations (2.1) through (2.3) and (2.6) through (2.10) can be written in terms of two variables with the sign designators removed as follows

$$4\ M_b \geq 2 \qquad (2.1a)$$

$$8\ M_a \geq 6 \qquad (2.2a)$$

$$4\ M_a \geq 6 \qquad (2.3a)$$

$$2\ M_a + 2\ M_b \geq 2 \qquad (2.6a)$$

$$5\ M_a + 3\ M_b \geq 6 \qquad (2.7a)$$

$$2\ M_a + 2\ M_b \geq 6 \qquad (2.8a)$$

$$14\ M_a \geq 24 \qquad (2.9a)$$

$$11\ M_a + 3\ M_b \geq 24 \qquad (2.10a)$$

Note that the fictitious joint rotations can not physically

occur alone and therefore appear in this final formulation only in the formation of the combined mechanisms. The minimum weight design problem is then to minimize

$$\text{Min } f = 2\, L_a\, M_a + L_b\, M_b$$

$$\text{or} \quad \text{Min } f = 8\, M_a + 4\, M_b \tag{2.11}$$

subject to expressions (2.1a) to (2.3a) and (2.6a) to (2.10a), where the function (2.11) is the design minimum weight function.

It is here that the assumption of a linear weight and moment relationship discussed on page 4 becomes important. In the present linear form the problem of the minimum weight design becomes a relatively simple linear programming problem in two variables, M_a and M_b, where the object function is (2.11) and expressions (2.1a) to (2.3a) and (2.6a) to (2.10a) are the constraining functions.

This linear programming problem will be solved for M_a and M_b in section 3.1.

2.2 Rigid Bracket Region Determination

In the limit design problem of very large and deep ships, with deep web frame structures, the effect of web overlap and bracket influence becomes significant. It is customary [5, 14] in elastic design of ship frame structures using beam theory to assume that a certain portion of brackets and overlapping webs constitute a rigid region on the ends of each span as shown in Figure

3. This method called the "Span Point Method" has been shown by finite element calculations to slightly over extimate the extent of the rigid regions.

To calculate the length of the rigid section in a given element Lund [6] uses the expressions

$$f_I = S_{II} / \sin \alpha$$

$$f_{II} = S_I / \sin \alpha$$

where f_I and f_{II} are the lengths of a rigid section in beams I and II respectively and S_I and S_{II} are the overlapping web depths associated with each beam. The value α is the angle of intersection between the two beams as shown in Figure 4a. The angle is measured at the vertex of the overlapping webs on the beam ends and is formed by the lines parallel to the beam axes through that vertex. It is evident that in the most common case of normal beams butting flush against each other, the rigid length influence on each beam of the intersection is the depth of the adjacent beam. Where three or more beams tie in at the same point, the beams with the greatest depth govern in this effect. In this case the bracket influence has been neglected.

To compute the influence of each bracket leg, the approximation

$$t_1^b = d_1 /\{(h/d_1) + 1\}$$

is used [6]. As is shown in figure 4b., t_1^b is the computed rigid length in the bracket, d_1, is the bracket

leg length and h is the beam depth. The total rigid length of one end of a given beam is the sum of the effects of the bracket and the beam overlap c_i, Figure 4b.

In this study, when bracket influence is considered, intersecting beams will be normal or nearly normal and it is assumed that $c_l = f_I$.

It should be noted here that the preceeding expressions are based on known values of geometry which are not known at the start of the design process. Iteration from an assumed starting value is required.

The rigid region assumption takes on special significance in the limit design problem. The effect is to indirectly specify regions within the structural system in which plastic hinges may not form. This forces span ends toward their midpoints while the total external work rate part of the virtual work relation may or may not remain unchanged, depending upon the nature of the particular mechanism under consideration. In any event, the formulation of the virtual work relation becomes significantly more complex in all but the simplest beam mechanism, especially in the external work part of the expression. The variability of rigid length values through a successive iteration steps compounds the problem. Lund [5] side steps this problem by calculating the rigid extensions only once at the problem beginning

and holding them constant thereafter.

Rigid section examples examined in this study were calculated by hand with rigid extensions assumed to be the same as used by Lund.

2.3 Combined Mechanisms and Kinematic Admissibility

Two final considerations are very important in the plastic limit design calculations performed in this study. These are limitations which can restrict combining mechanisms as was illustrated in section 2.1 and conflict with the requirement that all mechanisms be kinematically admissible.

The upper bound theorem of limit design requires that all velocity fields be kinematically admissible. This is a requirement that deserves special attention in formulating the limit design problem. For the velocity field in a plastic collapse mechanism to be kinematically admissible the velocity in the direction normal to the axes of the elements, u must be much larger than in line velocity, w. Figure 5 illustrates that only slight increases of the beam lengths are required to move point b a substantial distance laterally through beam rotation to point b' in the pictured mechanism.

Martin [22] states that the hinges of joint rotation mechanisms must form at the joint and not some finite distance out. This point is generally overlooked in the literature, but is of utmost importance if the

joint mechanism is to be utilized in forming combined mechanisms. The source of this problem is the requirement that the displacement fields of all limit design mechanisms be kinematically admissible. If a multi-element joint is rotated and if its hinges are located anywhere but at the center of rotation, kinematic admissibility as defined previously will be violated. Plastic hinges in addition to the joint mechanism hinges will be required before the mechanism can rotate.

The assumption of a rigid bracket region conflicts with the use of joint rotation mechanisms since the rigid regions permit hinges at their extremes away from the rotation center. Therefore, while the rigid joints can certainly be used as components of mechanisms, they cannot be considered as independent mechanisms. This all but eliminates linear combinations of mechanisms as discussed in section 2.1, if joint rotations are required to form the combination. When combined mechanisms are formed they require the formation of a very large number of plastic hinges with correspondingly high values of internal energy dissipation, reducing the probability that such a mechanism would contribute to collapse. In this study, combined mechanisms were considered only when they consisted of seven or less plastic hinges.

The use of the rigid bracket region assumption

can be one final source of trouble. If the rigid extensions are large and angles between hinge points are large, the proposed mechanism may be kinematically inadmissible. In Figure 6 which approximates the rigid bracket in a bilge region, u and w are of the same order of magnitude and the mechanism is inadmissible as is. Were hinge c moved farther to the right the mechanism could become admissible. How far the hinge must be moved before an admissible mechanism will be formed is a question of judgement. For this study, it was generally required that the height of the elevated hinge, h, be less than twice the distance from the corner below it to the next plastic hinge for the mechanism to be considered kinematically admissible. It is assumed that if this requirement is met, the value of α will be relatively small.

The rigid extension assumption is likely to be of greatest importance in tanker web frames which do not have cross ties. These frames will have very deep webs and correspondingly large overlap regions. One frame of this type was examined. In frames which have cross ties, the web depths will be less. In the transverse frames which had cross ties, the rigid extension assumption was not used. In frames of this type, plastic hinges were permitted anywhere in the beam element.

2.4 Summary

In this study, the upper bound theory of limit design is used to develop a system of linear relations which must be satisfied if the design structure is to safely carry the prescribed loads. A linear weighting or object function is formulated based on the lengths of the respective structural elements. These linear relations combine to form a linear programming problem which may be solved to find the minimum required plastic moments of the various structural elements in the system.

It is assumed that the plane frameworks of beams are made of materials with elastic, perfectly plastic moment curvature relations. It is also assumed that transverse shear and axial forces do not influence the plastic yielding of the beams.

Beam, panel and fictitious joint rotation mechanisms are considered in the formulation of linear, principal of virtual work, constraint relations as well as many combined mechanisms. Combined mechanisms which are very complex and fundamental sway mechanisms are not considered.

The assumption of rigid extensions in the area of beam overlap and brackets used in some elastic designs is considered in one case. This assumption disqualifies the use of fictious joint mechanisms in combining fundamental mechanisms on grounds of kinematic

inadmissibility. Mechanisms involving these rigid regions must be carefully checked for kinematic admissibility. The rigid extensions, where used, are calculated only once at the start of the design sequence.

3. OPTIMIZATION TECHNIQUE

3.1 Linear Programming

It was stated in the preceeding chapter that the linear equilibrium constraint relations and the minimum weight object formed a linear programming problem.

The general linear programming problem can be described as follows

> Given a set of i linear inequalities or equations in j variables, we wish to find non-negative values of these variables which will satisfy the constraints and maximize or minimize some linear function of the variables [30].

Mathematically this can be written minimize (or maximize) the linear objective function,

$$f = \sum_{j=1}^{N} c_j x_j \qquad (3.1)$$

subject to the linear equality constraints

$$\sum_{j=1}^{N} a_{ij} x_j = b_i \qquad i = 1, \ldots. \qquad (3.2)$$

and

$$x_j \geq 0 \qquad (3.3)$$

The values of a_{ij}, b_i, c_j are assumed to be known constants. Other notations, expecially vector forms, also appear in the literature. The use of inequalities is not entirely prohibited in equation (3.2) [29].

the use of slack variables permits the inequality

$$\sum_{j}^{N} a_{ij}x_j \geq b_m \tag{3.4}$$

to be expressed as

$$\sum_{j}^{N} a_{ij}x_j - x_{N+1} = b_m \tag{3.5}$$

$$x_{N+1} \geq 0 \tag{3.6}$$

The use of a plus sign in front of the slack variable would signify that the relation was of the less than (<) form.

Any set of x_j which satisfies the constraints (3.2) is called a solution to the linear programming problem. Any solution which satisfies the non-negativity restrictions of equation (3.3) is called a feasible solution. Any feasible solution which optimizes the objective function is called an optimal feasible solution. Generally, there will be an infinite number of feasible solutions to a linear programming problem. From all of these feasible solutions, one must be found which optimizes the objective function.

Linear programming problems which involve only two variables can be solved graphically. The graphic solution to these two variable problems can provide a great deal of insight into the more general linear programming case with any number of variables.

In Chapter 2, a simple portal frame was

examined and a system of linear constraining inequalities was developed which gave upper bounds on the plastic limit moments required to make the structure safe. These inequalities in two variables with the minimum weight objective function form a linear programming problem which is illustrated graphically in Figure 7.

The solid lines represent the various constraint functions. The dotted lines are the parallel lines of constant weight of the object function. The solid lines with cross-hatching attached bound the region of feasible solutions (or the acceptable design region). This is the boundary of the region where all inequality constraint relations are satisfied. The optimum design is found at the point, or points, where the object function

$$\text{Min } f = 8M_a + 4M_b$$

has its lowest value while just touching the boundary of feasible solutions. The figure shows that the variables of the optimal solution are approximately $M_a = 1.8$ and $M_b = 1.1$. To find the exact values, simultaneous solution of the equations of the two intersecting lines, equations (2.8a) and (2.10a) is required. Restated, these are

$$2\,M_a + 2\,M_b = 6$$

$$11\,M_a + 3\,M_b = 24$$

Solution of these equations gives exact values

of $M_a = 1.875$ and $M_b = 1.125$. These values represent exact, upper bound solutions to the minimum values of plastic limit moment required to make the design structure of Figure 1, safe under the prescribed loads. Note that no factors of safety have been applied at this point.

In this case the optimum solution was defined to be at single vertex formed by two boundary lines. It is also possible that the objective function could have passed through two vertices as well as all points on the line between them. In cases of this type, it is said that the linear programming problem has alternative optima. This condition occurs when the objective one of the extreme points is optimal. Unfortunately these methods do not tell which one [29].

Other methods must be used to solve linear programming problems. There is presently no method available which will find an optimal solution to a linear programming problem in a single step. All of the proceedures for solving these problems are, therefore, iterative. If the number of constraints is large, it becomes very difficult to find any feasible solution and still more difficult to find an optimal solution.

3.2 The Revised Simplex Method

The revised simplex method of solving linear programming problems is a revision of the simplex method

derived by Dantzig in 1948. The term "simplex" has nothing to do with the technique as it is now used. To properly introduce the revised simplex method it is helpful to start with a short discussion of the simplex method. Both techniques are algebraic iterative procedures which will solve exactly any linear programming problem in a finite number of steps, or give an indication that the solution is unbounded. There are a finite number of extreme points and if there is an optimal solution, one of the extreme points is optimal. The simplex method is a technique which moves from a given extreme point to another adjacent and more optimal extreme point, along the boundary of the feasible solution region. Of all adjacent extreme points, the one chosen is the one that gives the greatest decrease (or increase) in the object function. At each iteration, the simplex method defines whether or not the associated extreme point is optimal and if not, it defines the next extreme point to be examined.

Complete discussions on the simplex and revised simplex methods of linear program are by nature quite lengthy. It is not the purpose of this study to examine in depth the mathematical rational for either of these methods. Detailed information on theory, development and application can be found in References

[29 - 31].

The simplex method has two phases. The first phase finds a first basic feasible solution which satisfies the condition that the m constraints intersect and that there are only m nonzero variables x_j. These x_j variables are called the variables of the basis. The remaining (N - m) of the variables x_j are equal to zero and called non basic. At the end of the first phase the results are structured in standard canonical form. In order to satisfy inequality (3.3) all values of b_i must be greater than zero [8]. In practice this may require the introduction of artificial variables.

When the canonical form of the first phase is obtained, the second phase begins. Given any basic feasible solution an iteration step is made toward an optimal feasible solution by changing a single vector in the basis. In the revised simplex method, the criteria used to determine the vector to enter and leave the basis are identical with the simplex method. Changes in the basic feasible solution always lead back to the canonical form with the variables x renumbered. The essential element that allows progression from each basic feasible solution is the explicit knowledge of the representation of the vectors not in the current basis in terms of the basis vectors.

The main difference between the simplex method

and the revised procedure is that in the simplex method all elements of the simplex tableau are transformed by means of elimination formulas, while in the revised simplex method only the elements of an inverse matrix need by transformed using these formulas.

Since its development the revised simplex method and in particular a variation which employs the product form on the inverse, has been used in the larger high-speed computers [29].

There are several reasons for the current widespread use of the simplex method. For problems with sparse coefficient matrices, computation is reduced. The revised procedure always deals with original coefficients. The amount of new information required to be stored is less than in the simplex method, since only the inverse and the solution vector must be stored and not the complete simplex tableau as in the simplex method.

To formulate the computational procedure for the revised simplex method equations (3.1) through (3.3) will first be restated in vector form. In the general linear programming problem a minimum value of the object function

$$\underline{c}\ \underline{X} \tag{3.5}$$

is sought subject to

$$\underline{A}\ \underline{X} = \underline{b} \tag{3.6}$$

and

$$\underline{X} \geq 0 \tag{3.7}$$

letting the matrix $\underline{B}$ of the basis of m-dimensional vectors correspond to the first m vectors of $\underline{A}$, such that

$$\underline{B}\ \underline{X}_0 = \underline{b} \tag{3.8}$$

$$\underline{X}_0 \geq 0 \tag{3.9}$$

where $\underline{X}_0 = (x_{10}, x_{20}, \ldots .\ x_{mo})$ is the solution vector. A basic feasible solution is given by

$$\underline{X}_0 = \underline{B}^{-1}\ \underline{b} \tag{3.10}$$

The linear combinations of all the vectors of $\underline{A}$ in terms of $\underline{B}$ can be found by

$$\underline{X}_j = \underline{B}^{-1}\ \underline{P}_j \qquad j = 1,2,\ldots n \tag{3.11}$$

where $\underline{X}_j$ is a column vector and $\underline{P}_j$ is a set of linearly independent vectors associated with each extreme point and can be expressed

$$\underline{P}_j = x_{1j}\ \underline{P}_1 + x_{2j}\ \underline{P}_2 + \ldots . + x_{mj}\ \underline{P}_m \tag{3.12}$$

The initial basic feasible solution is found by setting $\underline{X}_j = \underline{P}_j$.

For a given iteration solution the value of the objective function is

$$z_j = c_1\ x_{1j} + c_2\ x_{2j} + \ldots . + c_m\ x_{mj} \qquad j = 1, 2, \ldots ., n \tag{3.13}$$

where c_i are cost coefficients of those vectors in the basis. Converting to vector form $z_j = \underline{c}_0\ \underline{X}_j$ where $\underline{c}_0 = (c_1, c_2 \ldots .\ c_m)$ is a row vector.

From equation (3.11) z_j can be written in the

form

$$z_j = \underline{\pi}\ \underline{P}_j \tag{3.14}$$

where $$\underline{\pi} = \underline{c}_0\ \underline{B}^{-1} \tag{3.15}$$

$\underline{\pi}$ is an m-dimensional row vector called the multiplier vector. The individual values π_i in the vector are called the simplex multipliers. Given the value of $\underline{\pi}$ for a given basis $\underline{B}$, the corresponding z_j can be calculated. A vector $\underline{P}_j$ not in the basis is evaluated by computing $\underline{\pi}\ \underline{P}_j - c_j = z_j - c_j$.

The information needed to proceed from one feasible solution to the next improved feasible solution is the original data consisting of $\underline{A}$, $\underline{b}$ and $\underline{c}$ and equations (3.10), (3.11) and (3.15), if for each feasible basis $\underline{B}$, $\underline{B}^{-1}$ is known explicitly [29]. This concept is fundamental to the revised simplex method of solving linear programming problems.

The revised simplex linear programming procedure has been applied by large numbers of users to a wide variety of problems with efficient results. Most often, it can be treated as a "black box" operation with only general knowledge on the part of the user. It forms a part of most computer application libraries and is available at little or no increase in cost over the basic computer usage fees [8].

The subroutine called in this study was the Mathematical Programming System Extended (M.P.S.X.) as

installed on the IBM 370 Model 168 computer. Specifics on the use of the subroutine are outlined in the appendices.

4. DISCUSSION OF RESULTS

4.1 Single Cross-Tie Frames

The results of the twenty-five frames evaluated in Appendix A varied significantly. The lightest frame had a cross-tie height of 12.0 meters (as did all of Lund's frames) and a side tank breadth of 12.0 meters. Lund's lightest frame had a side tank width of 16.0 meters. The weights of the lightest limit design frame were 41.47 metric tons and 48.50 metric tons for factors of safety of 1.5 and 2.0 respectively. All of Lund's frames are based upon a factor of safety of 1.5 and his lightest structure weighed 42.1 metric tons.

The heaviest limit design frame structure had a cross-tie height of 10.0 meters and a side tank width of 20.0 meters. The weights of this frame were 56.08 metric tons and 63.45 metric tons for factors of safety of 1.5 and 2.0 respectively.

The total frame weights of the elastic design structures and the limit design structures are within 10.0 metric tons for all frames having factors of safety of 1.5.

The distribution of frame weights resulting from the different design methods is significant.

Member proportions were kept constant through both elastic and plastic limit design optimization,

excluding plate flange areas. This procedure allows direct comparison of the relative weights of members with the same proportions by comparison of top flange areas. Comparisons of beam proportions are made using a factor of safety of 1.5 for the limit design results unless stated to be otherwise.

The limit design results show the size of Member #5, Figure A.1, to be larger than the Lund results for almost all frames considered. The limit design results with factors of safety of 1.5 gave top flange areas of 124 square centimeters in the lightest beam case and 283 square centimeters in the heaviest beam case. Lund's heaviest lower side tank beam had a top flange area of 127 square centimeters and his lightest had an area of 65 square centimeters. As might be expected the heaviest beams were those with the longest spans. The limit design lower side tank beams have roughly twice the cross sectional area of the elastic designs with the same side tank breadth in all cases. For example, Lund's frames with 16 meter side tank widths gave values of 92 and 127 square centimeters for top flange area. For similar frames the limit design results gave values of 198, 200, 203, 205, and 207 for top flange areas of the lower side tank beams. The dimensions of the Lund elastic frames may be found in the second rows of Tables 3 and 4 of Reference [5].

The model used in the limit design study considered all loads acting on the structure. Lund does not discuss his model is detail. It is not known if he accounted for the influence of loads on the rigid extensions in his beam analysis. If loads in the rigid regions were neglected in the elastic designs, this could account for his smaller required size for the lower side tank beam.

With one notable exception other beam elements were generally slightly lighter for the limit design case. The values calculated for the dimensions of Element #7 were much less in the limit design results. Representative values of from 69 to 101 square centimeters were calculated for the top flange areas of the limit design frames versus a range of 134 to 253 for the elastic designs. In general, the elastic designs were roughly twice the weight of the plastic limit designs for this beam element (i.e. Element #7). This difference may have again been caused by unknown differences in models.

All comparisons, so far, have been based on identical values of factors of safety of 1.5 for elastic and plastic limit designs. Figure A.5 shows that when a factor of safety of 2.0 is applied to the plastic limit designs, the total frame weights become higher than all but one of the frames designed by elastic methods. The

effect of increases in factors of safety on individual beam elements is illustrated in Figure A.4 for one beam element for the various plastic limit design frame geometries. Values of net frame area A_T are plotted against factors of safety for Member #1 of all frames evaluated. The dots represent values for the elastic designs of Lund. The increase in net area is seen to be nearly linear. On this figure only the line representing frames having cross-tie heights of 12.0 meters should be compared with the elastic design values since all elastic designs had 12.0 meter cross-tie heights.

Plots for other beam elements would be similar to Figure A.4, but there could be up to 25 different lines if each of the frames considered had different values of A_T for the element considered.

As would be expected from looking at Figure A.5, the weights of the individual limit design beam elements are in general heavier than Lund's results when a factor of safety of 2.0 is applied to the limit design results.

The design effort required in evaluating the twenty-five frames discussed here was minimal. Total computation cost was $91.65. This results in a computation cost per frame of $3.66. The limit design problem was greatly simplified by using the assumption that the influence of brackets and beam overlap could be neglected.

Lund did not use this assumption. His elastic designs all assumed rigid joint regions. The elastic design computation time is much larger than for the plastic designs. Lund [5] cites CPU-time of 50 to 60 seconds on a UNIVAC 1108 for design of one frame structure. The limit design solutions of Appendix A required approximately 8 seconds in CPU-time for one frame structure on the IBM 370 including M.P.S.X. linear programming solutions.

4.2 Zero Cross-Tie Frame with Rigid Corner Assumption

The results obtained in Appendix B were for a single frame based on the geometry of Lund's frame numbered OC22, 49.2t. The values of total frame weight for a number of factors of safety are plotted in Figure A.6. The weight of the elastic parent frame is plotted as a point at a factor of safety of 1.5. From the figure it may be seen that the weight of the limit design frame calculated with a factor of safety of 1.5 is twenty per cent lighter than the elastic frame. At a factor of safety of 2.0, the limit design structure weighs approximately one half ton less than the elastic design structure.

In the top line of Table 4, Reference [5], Lund lists the dimensions of his elastic design frame. Table A.2.4 lists values for the limit design frame for factors of safety of 1.0 to 2.5 in 0.5 increments. If the limit design dimensions with a factor of safety of 1.5 are

compared to the Lund results, it will be seen that the lower side tank girder, Member #2, is again substantially larger in the limit design results than in the elastic design results. Using top flange areas as a measure of relative weight, the limit design beam has a value of 192 cm^2, while the elastic design beam has an area of 167 cm^2. This gives a relative difference in beam weights of approximately 15 per cent.

As mentioned previously, it is not known if Lund accounted for the influence of loads on the rigid extensions. The limit design calculations accounted for all loads on the structure including the rigid regions. If Lund did not do this in his design model, his results could be lighter than the plastic limit design values.

With the exception of Member #5, in the side tank top, all of the remaining limit design beam elements were about half the weight of their elastic design counterparts. Member #5 in the side tank top was about one fourth the weight of its elastic design counterpart.

The limit design calculations done in Appendix B were done by hand as opposed to Lund's computer approach. The hand operations were performed to allow close inspection of each mechanism. The results were taken to the computer for solution of the linear programming problem. Computation time was approximately 6 seconds. Again, the CPU-time for Lund's solutions averaged 50-60 seconds per

frame structure. The time required to do the limit design hand calculations was approximately 30 hours on the first time through. This time could be reduced to about four hours for successive runs by using the models already developed.

These solutions could also be adapted for computer solution as was done in Appendix A using a model similar to the one used in the hand calculations and a heavy reliance upon computer conditional tests. The conditional tests would be required to check the possible mechanisms for kinematic admissibility.

5. SUMMARY AND CONCLUSIONS

An analytical study has been undertaken to investigate the use of the upper bound theorem of limit design in the design of transverse web frames in ships. Frames have been developed using limit design with results compared to frames of elastic design.

In Appendix A the influence of beam overlap and brackets was neglected. This assumption was not made in the elastic design case. The results based on this assumption were frame structures of roughly equal weight to the elastic frame designs with some frames less than the least weight elastic designs if the same factor of safety is applied in both methods. For a factor of safety 2.0 the plastic limit design structures are approximately 10 per cent heavier.

It is significant that while the overall structural weights are similar in this comparison, the distribution of weight is somewhat different. Frame structures designed by elastic methods have their largest members in one location while structures designed by the plastic method have their largest members in another location.

In Appendix B the effect of beam overlap and bracket influence was idealized by assuming portions of these regions to be infinitely rigid.

Structural weight calculated using this assumption was approximately equal to the elastic design if a factor of safety of 2.0 was applied to the limit design solution. If a factor of safety of 1.5 is applied to the limit design solution, its result gives a twenty per cent reduction of weight over the elastic design.

A specific factor of safety was not specified for the plastic designs but rather a range of values was examined. The important advantage of limit design is that it allows the designer to define exactly the collapse moments for a given set of loading conditions. Factors of safety may be applied to these known quantities as required.

In the case of the elastic design methods, factors of safety are defined somewhat arbitrarily without specific knowledge of when collapse will occur. The knowledge of specific collapse moments allows structures designed using plastic design methods to be much more efficient.

The geometry of individual beam cross sections was not optimized or allowed to vary from the Lund proportions in this study. This was to permit direct comparison with the Lund designs. The limit design solutions could possibly have been made much lighter by use of more efficient beam cross section geometry.

In the case where the assumption of rigid

corners was made, no attempt was made to calculate structural features within the rigid assumption zone. If this assumption is to be used, corner scantlings will have to be developed by separate analysis.

Neglecting other possible advantages, the design costs using plastic limit design are very cheap. A design solution for a rather sophisticated tanker web frame can be had for very few dollars. This allows a number of different geometries to be examined as in Appendix A for the price of a single design found by elastic methods.

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LIST OF FIGURES

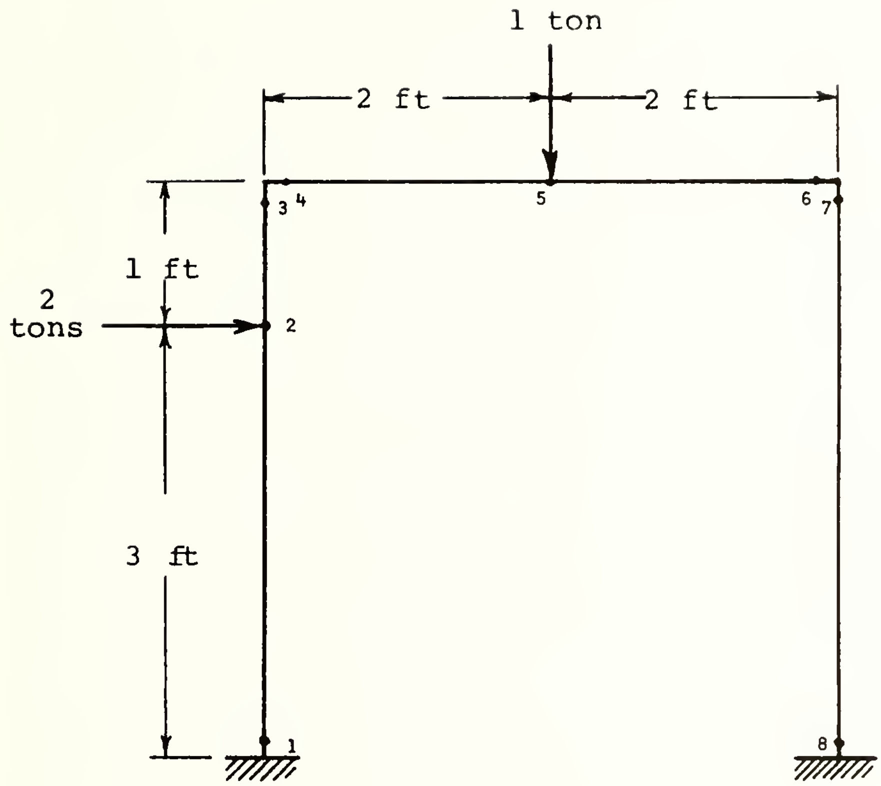


Figure 1. One bay portal frame.

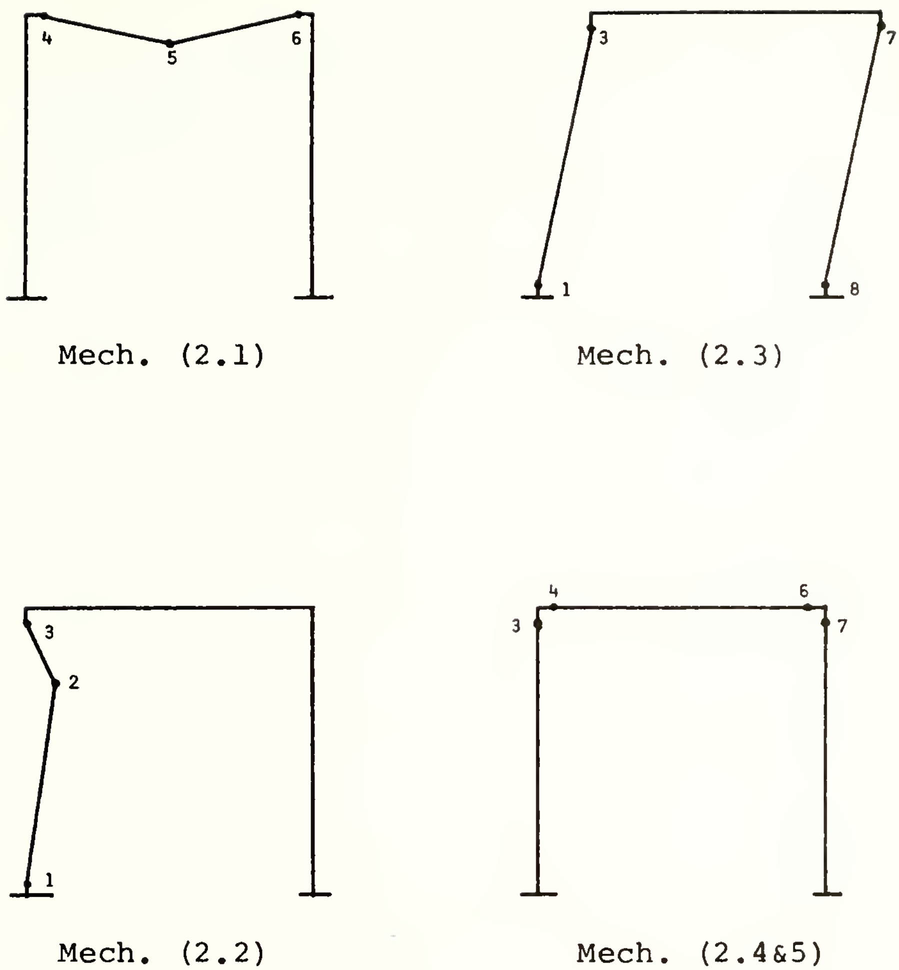


Figure 2. Fundamental collapse mechanisms of portal frame in Figure 1.

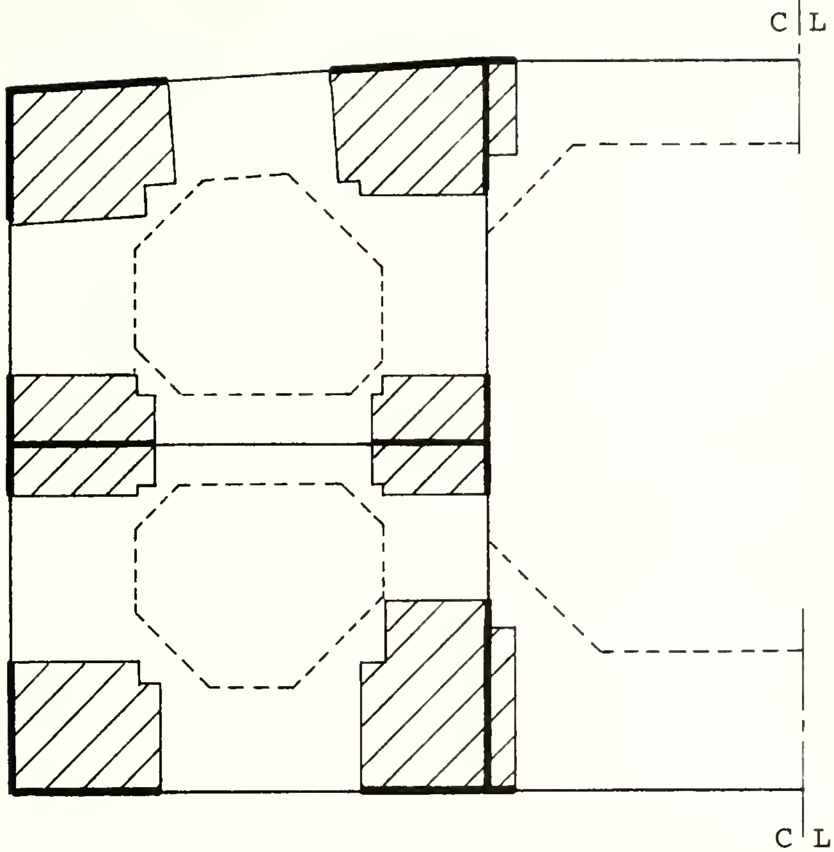


Figure 3. Tanker transverse web frame model with rigid extensions due to beam overlap and bracket influence.

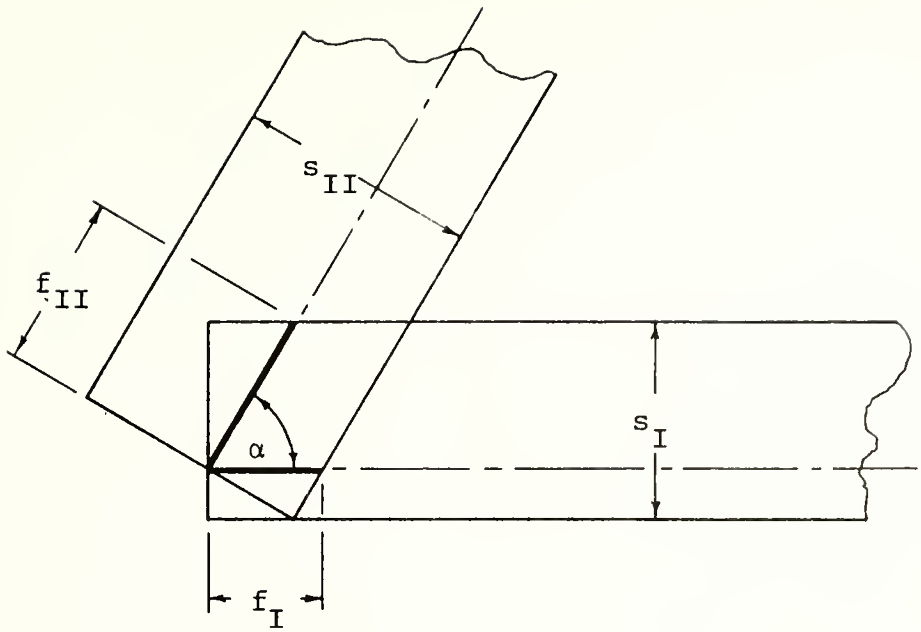


Figure 4a. Corner model for calculation of rigid extensions due to beam overlap.

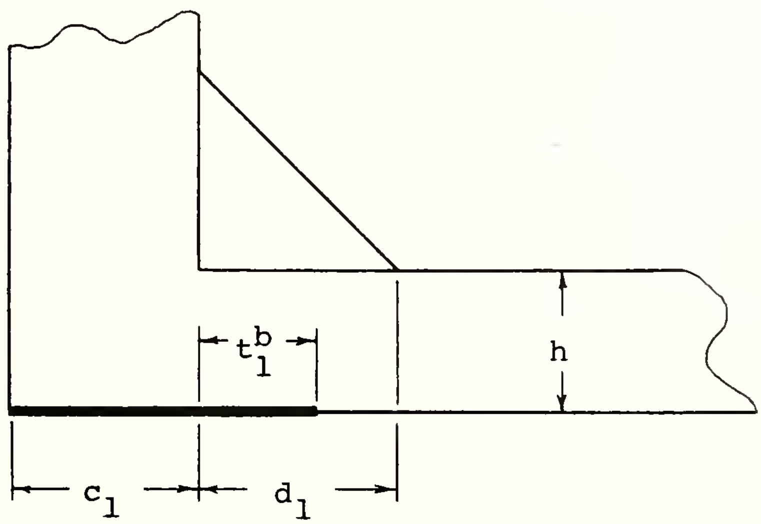


Figure 4b. Corner model with bracket for calculation of rigid extensions.

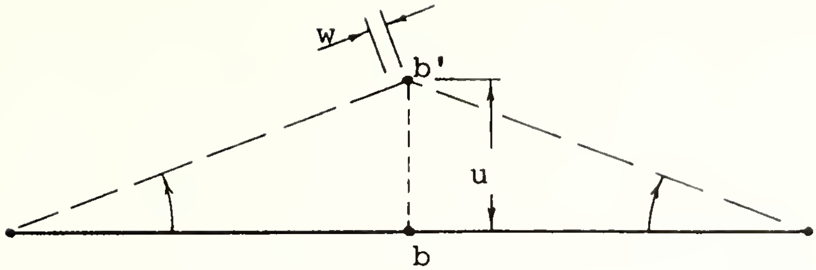


Figure 5. Kinematically admissible collapse mechanism.

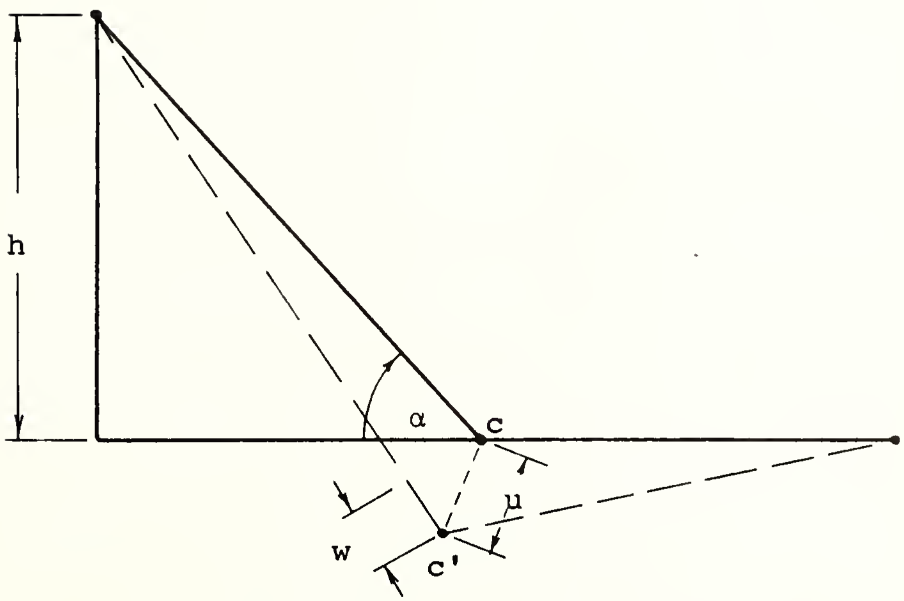


Figure 6. Kinematically inadmissible collapse mechanism with w approximately equal to u.

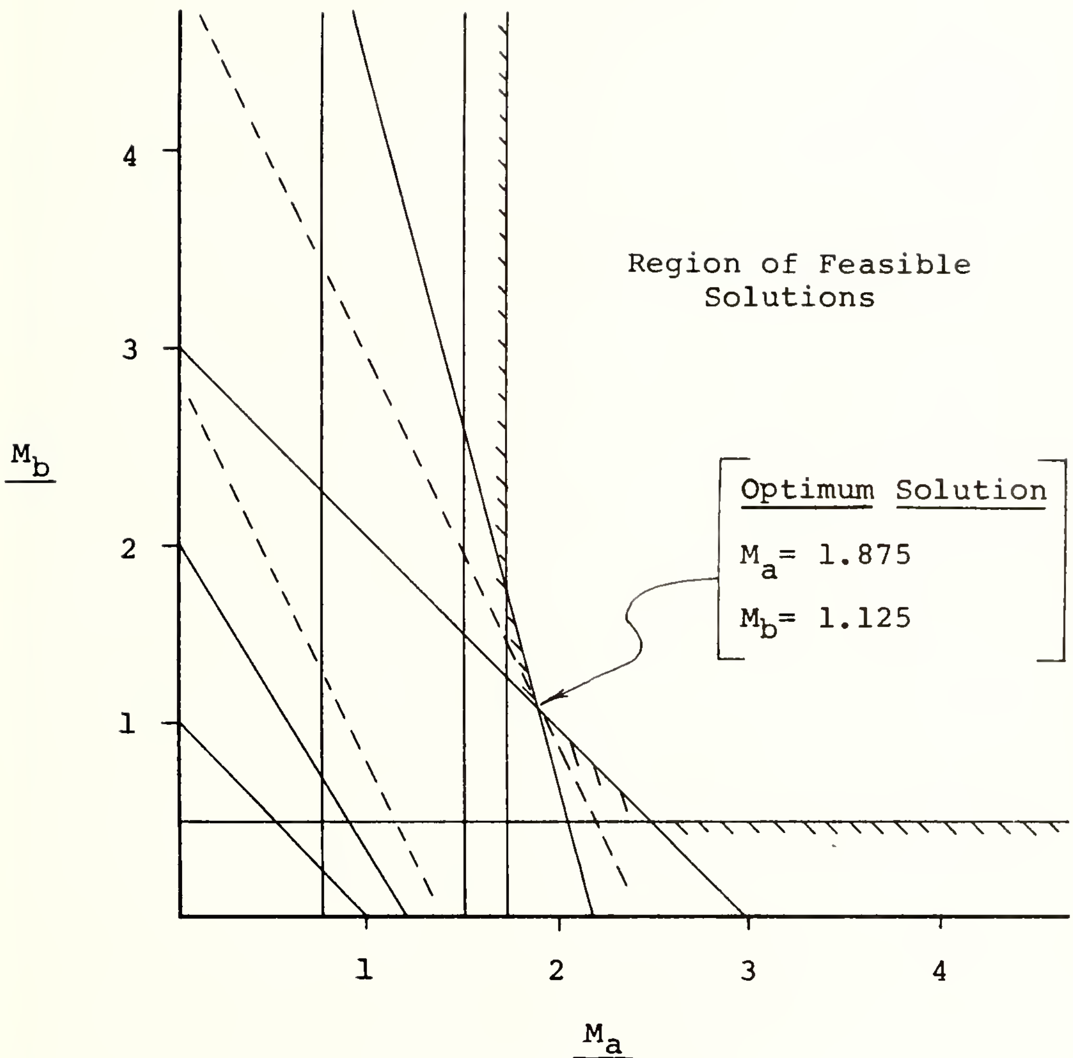


Figure 7. Graphical solution to linear programming problem of portal frame example.

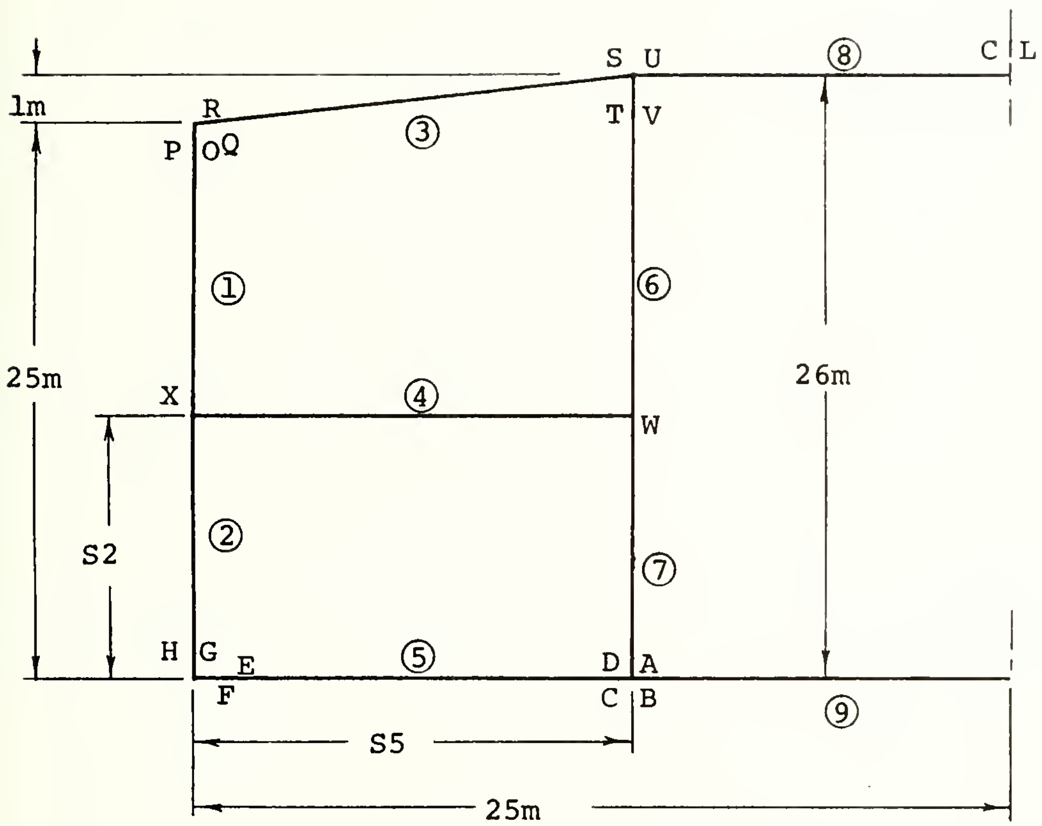
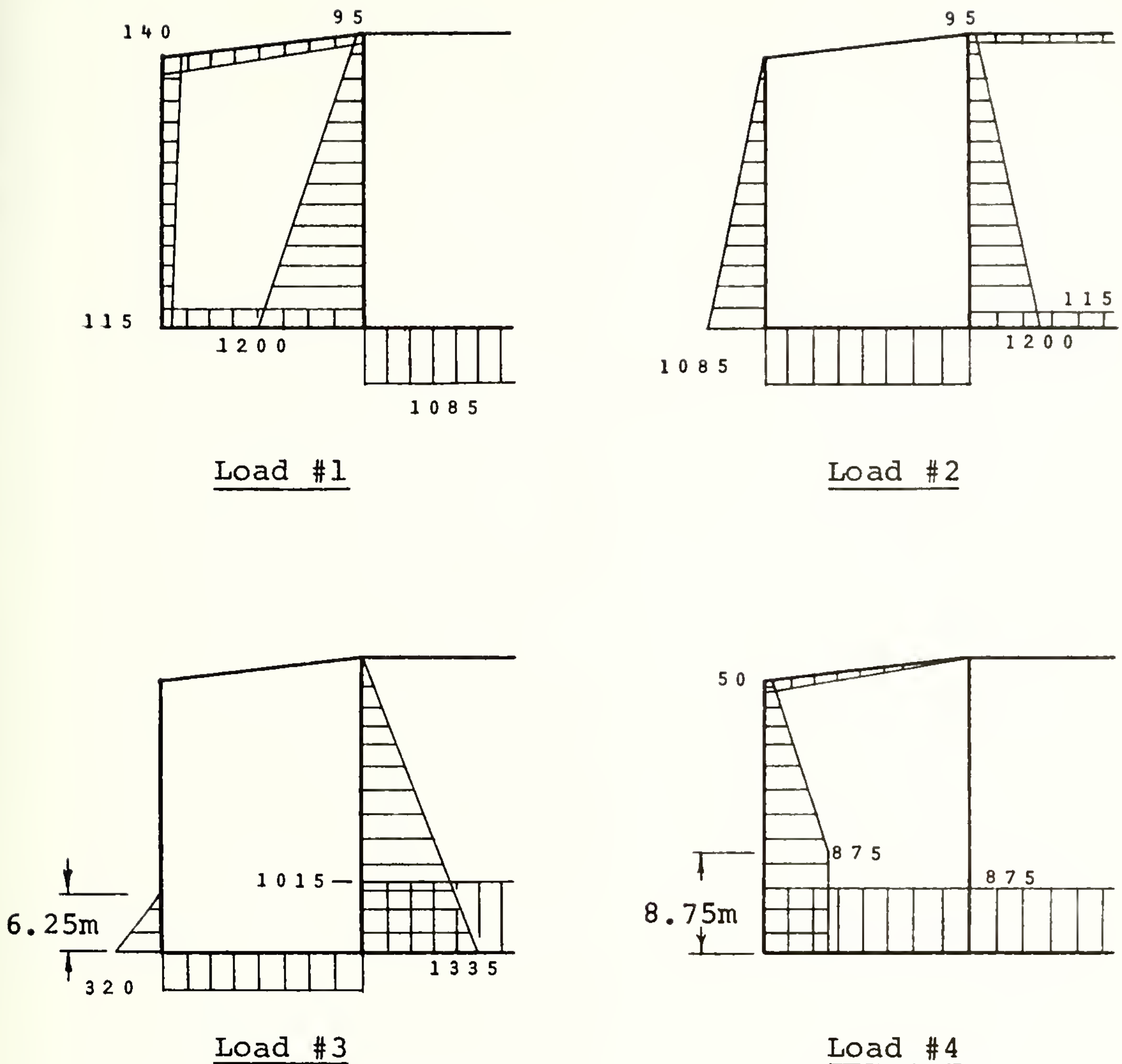


Figure A.1. Tanker web frame model with one cross-tie.



(Loads in kp/cm)

Figure A.2. Loading conditions considered in tanker web frame design.

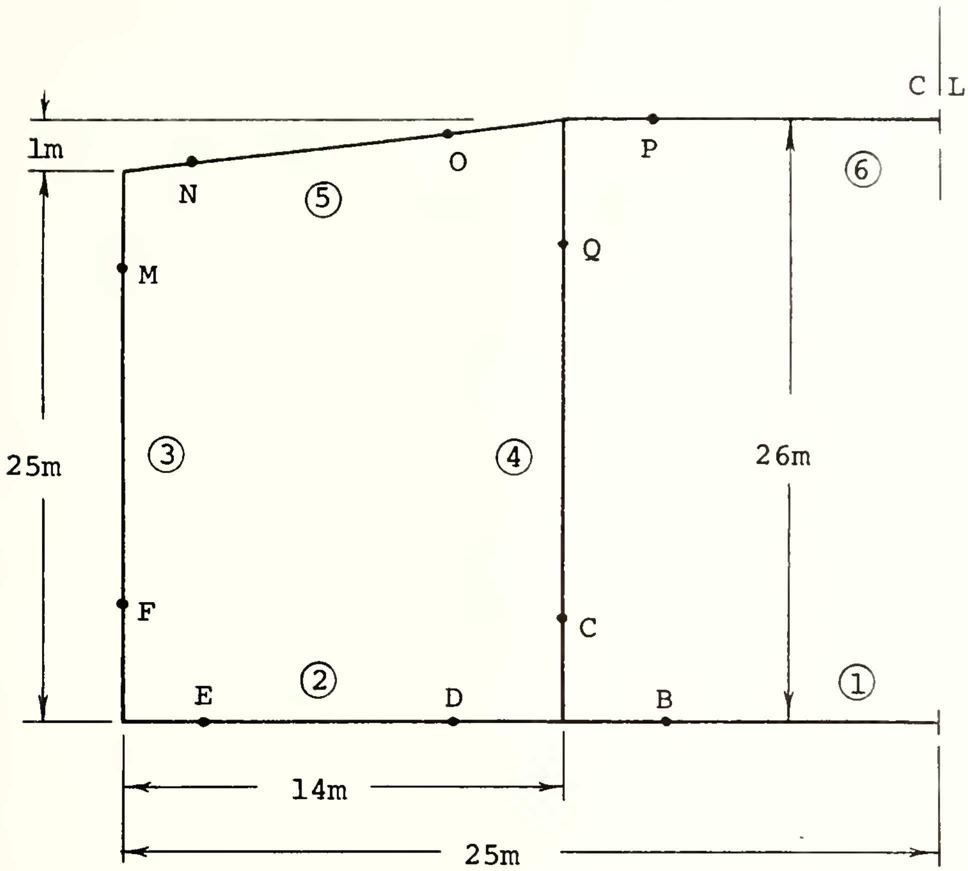


Figure A.3. Tanker web frame model without cross-ties and with rigid extensions.

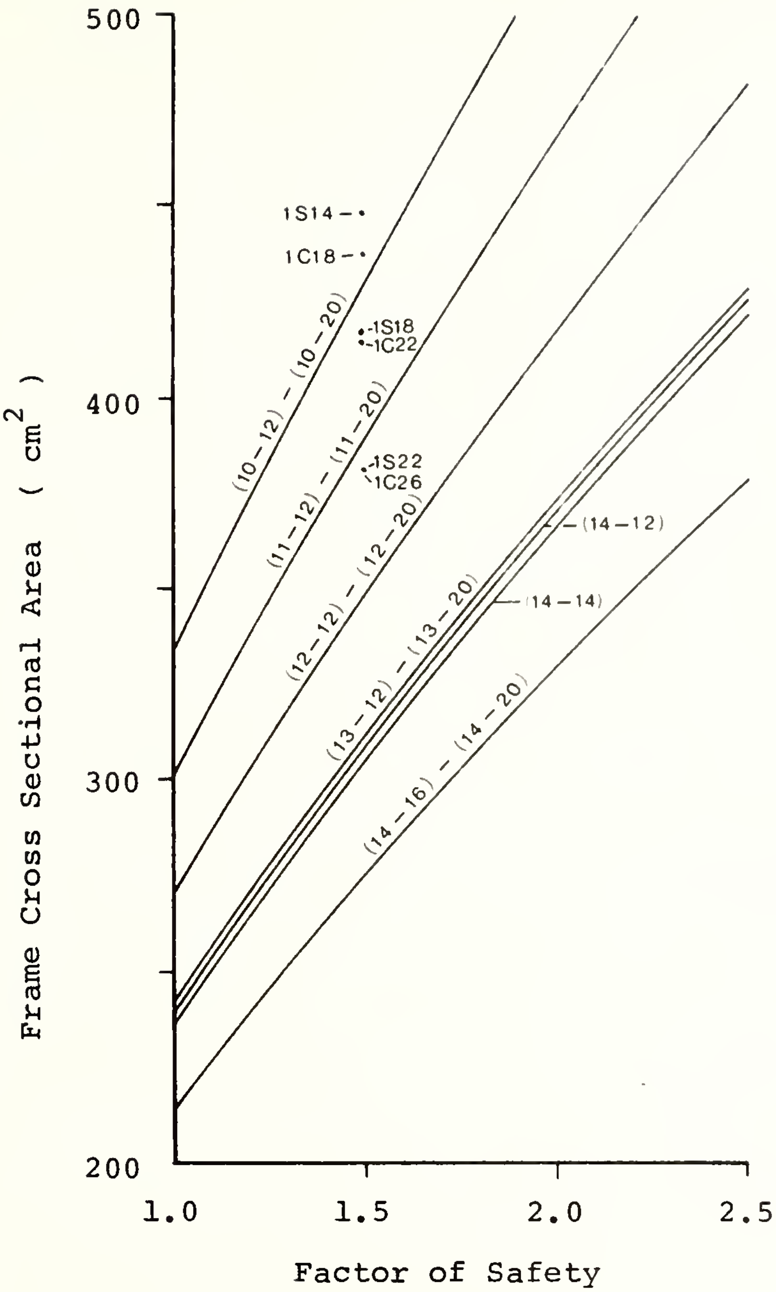


Figure A.4 Frame cross sectional area vs. factor of safety for beam Element #1 of tanker web frame with a single cross-tie.

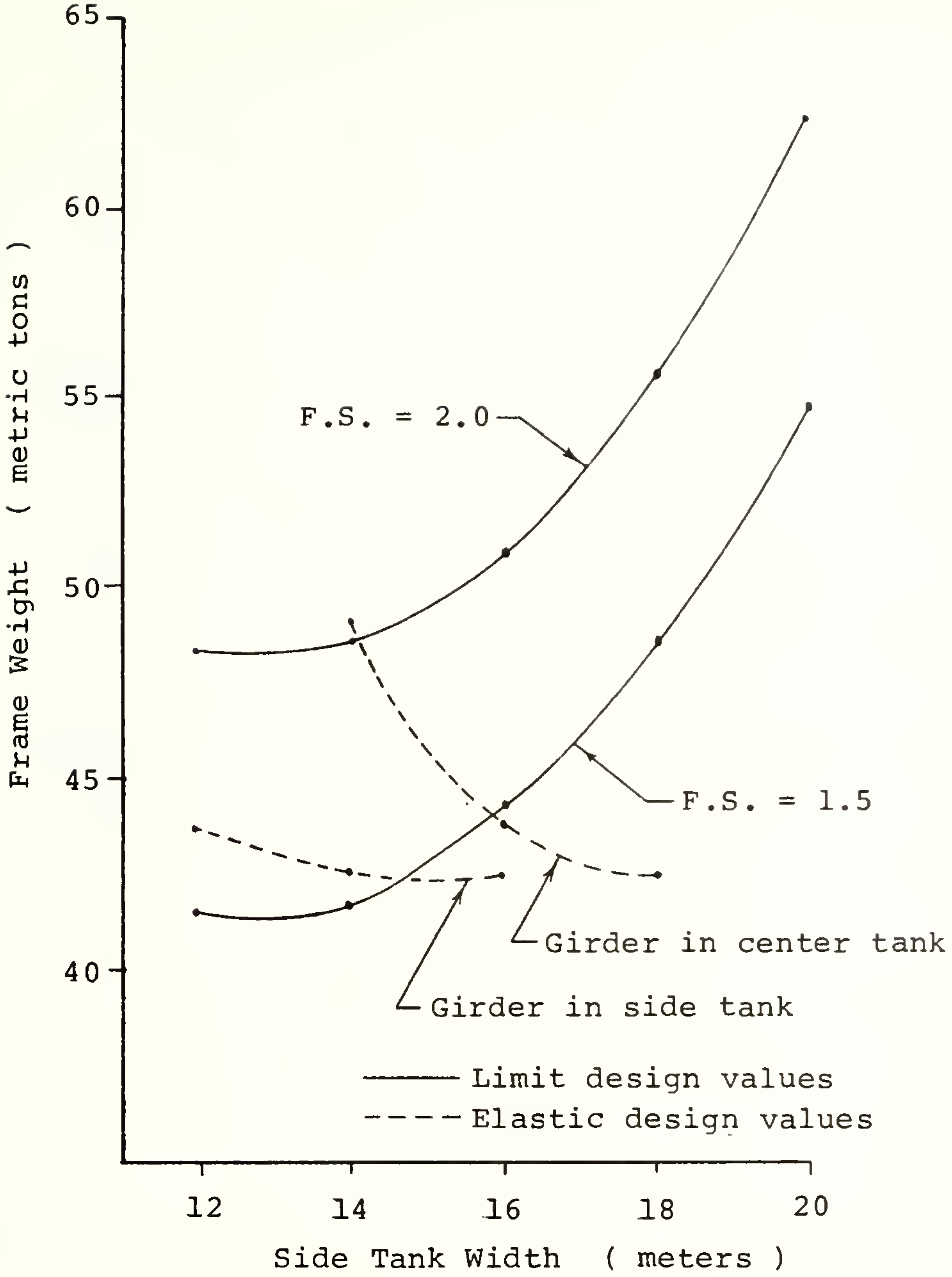


Figure A.5 Various frame weights for limit design and Lund elastic frames having a single cross-tie located twelve feet above the base line.

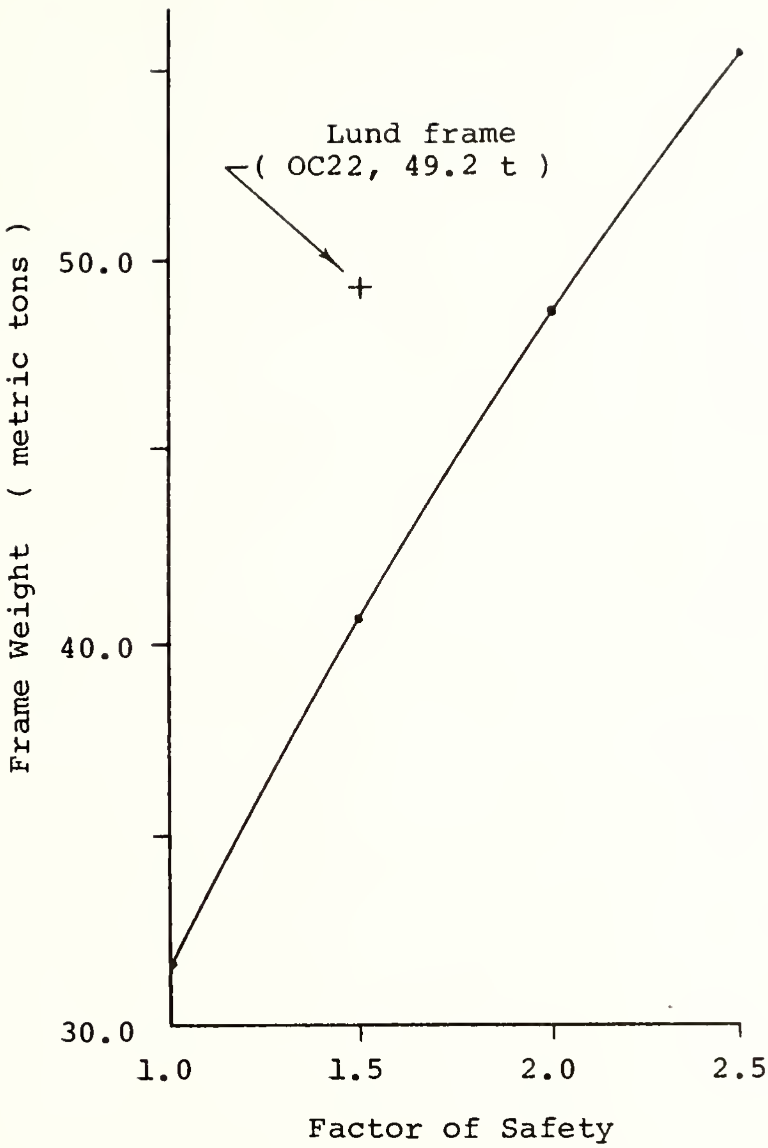


Figure A.6 Weights of zero cross-tie tanker web frame with rigid joints for various factors of safety.

APPENDIX A - TANKER FRAME COMPARISON

Modern supertankers are designed with large variations from ship to ship in the internal primary stiffening system. The number of cross-ties varies from zero to three. Structures may be designed with or without longitudinal side girders. Bottom center girders may be low or high. When compared on the basis of weight to strength ratios, some designs should be superior to others [5].

The present study comprises investigations of variations of a single transverse web frame configuration.

A FORTRAN computer program was developed to evaluate the limit moment values of a tanker webframe having a single cross-tie located in the wing tank. The vessel considered is identical to the one considered by Lund in Reference [5]. The vessel is a typical supertanker with the following principal dimensions:

Length	L_{pp} = 320.0 m
Breadth	B = 50.0 m
Depth	D = 25.0 m
Draft	d = 19.5 m
Camber (all in side tank)	= 1.0 m
Spacing of transverse frames	= 5.0 m

The basic geometry of the frame studied is

shown in Figure A.1. The program was developed specifically to evaluate variations of this frame, but it could be made more general without extensive modification. As written, the required input values are the cross-tie height above the base line S2 and the wing tank breadth S5, both values being in centimeters. All other values are either written into or calculated by the program as needed, based on the parent vessel geometry and the four loading conditions prescribed by Lund [5] and to be described later in this appendix. System lines are assumed to be formed by the inside surface of the shell plating and the center tank face plating of the longitudinal bulkhead. It was found when beams were sized that plastic neutral axes for the various beam elements remained at or very near these system lines.

The possible influence of beam overlap and brackets was neglected with plastic hinges allowed to form anywhere along the lengths of individual beam elements. Combined plastic collapse mechanisms which are very complex and are unlikely to be actual collapse mechanisms have been eliminated from computation a priori to reduce computing costs.

The range of cross-tie heights examined is from 10 meters to 14 meters in one meter intervals. The range of side tank breadths examined is from 12

meters to 20 meters in 2 meter intervals. In all a total of twenty-five separate variations of the same single cross-tie web frame were examined.

The loading conditions applied were identical to those used by Lund in his elastic designs. The various conditions are shown in Figure A.2. The loading models as applied represent the following situations [5].

1. Empty center tank, cargo in side tanks, full draft with dynamic additions.
2. Empty side tanks, cargo in center tank, full draft with dynamic additions.
3. Empty side tanks, ballast in center tank, draft of 0.25 D.
4. Ballast in center and side tanks, draft of 0.35 D.

As stated previously, the program input variables are cross-tie height and side tank breadth, S2 and S5 respectively. The input format is free with the real values of S2 and S5 in centimeters listed in the same order shown here. The program sequence exits when a zero value for S2 is read so the last data card should have real values of zero and any other real constant listed consecutively.

The output values of the FORTRAN program include the coefficients of the internal energy dissipation portion of the limit design upper bound

theorem. These coefficients are presented as the values a_{ij} of the coefficient matrix $\underline{A}$ in the limit design problem constraint relation. The associated values of the external work rates in the principle of virtual work relations are also given as output. These work values represent the right hand side of the constraint equations $\underline{b}_i$.

The output information is all punched on cards to be used as input data in the linear programming problem to be solved by the M.P.S.X. subroutine. The format of the punched cards is quite important, but is not discussed here. A discussion of M.P.S.X. input format is given in the numerical routine appendix.

Finally, a punched card is inserted at the front of each output data deck listing the values of S2 and S5. This is to enable the user to identify the various output data sets when more than one frame is examined.

The program was checked for logical and numerical errors by comparing output values and hand calculation results on the web frame having S2 = 12.0 meters and S5 = 12.0 meters.

The complete listing of the program with sample data values is presented on the pages that follow.

FORTRAN Program for Limit Design of Tanker Web Frame Based on the Model in Figure A.1

```
// 'KINGHN',CLASS=A,REGION=250K
/*MITID USER=(M13516,P14908,,,TANKWEB)
//  EXEC WATFIV
//C.SYSIN  DD *,DCB=BLKSIZE=2000
$JOB
      DIMENSION A(99,9),W(99)
    1 READ,S2,S5
      IF(S2.EQ.0.)GOTO 2
C ELEMENT LENGTH EVALUATION
      S1=2500.-S2
      S7=S2
      S6=2600.-S7
      S4=S5
      S3=(1.+(S5**2))**.5
      S8=2500.-S5
      S9=S8
C LOAD 1 INFLUENCE
      PB=1085.
      A(10,9)=4.
      W(10)=PB*(S9**2)
      PC=115.
      A(11,5) =4.
      W(11)=S5**2)*PC/4.
      A(12,2) =1.
```

```
A(12,5)=3.
W(12) =W(11)
A(13,2)=1.
A(13,5)=2.
A(13,7)=1.
A(13,9)=1.
W(13)=W(10)/2.+W(11)
A(14,5)=3.
A(14,7)=1.
A(14,9)=2.
W(14)=W(13)
PH=115.
PP=140.
PX=(S2*(PP-PH))/(S1+S2)+PH
R=(PX-PH)/(S2+S1)
F=(PX-PH)/(4.*S2)-(R/2.)
B=(R*S2)/3.-(PH+2.*PX)/6.
C=S2*(PX+2.*PH)/12.
SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
A(15,2)=2.+2.*(SH/(S2-SH))
PM=SH*(PX-PH)/S2+PH
W(15)=(SH**2*(2.*PM+PH)+(S2-SH)**2*(PX+2.*PM)*SH/(S2-SH))/6.
A(16,2)=11+2.*(SH/S2)
A(16,3)=1.
W(16)=W(15)
F=(PP-PX)/(4.*S1)-R/2.
B=(R*S1)/3.-(PX+2.*PP)/6.
C=S1*(PP+2.*PX)/12.
SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
A(17,1)=2.+2.*(SH/(S1-SH))
PM=SH*(PP-PX)/(S1-SH)+PX
W(17)=(SH**2*(2.*PM+PX)*SH/(S1-SH)+(S1-SH)**2*(PP+2.*PM))/6.
```

```
A(18,1)=1.+2.*SH/(S1-SH)
A(18,3)=1.
W(18)=W(17)
PQ=140.
PT=95.
R=(PQ-PT)/S3
F=(PQ-PT)/(4.*S3)-R/2.
B=R*S3/3.-(PT+2.*PQ)/6.
C=S3*(PQ+2.*PT)/12.
SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
PM=(SH/S3)*(PQ-PT)+PT
A(19,3)=2.+2.*SH/(S3-SH)
W(19)=(SH**2*(2.*PM+PT)+(S3-SH)**2*(PM+2.*PT)*SH/(S3-SH))/6.
A(20,1)=SH/(S3-SH)
A(20,3)=2.+SH/(S3-SH)
W(20)=W(19)
PV=95.
PD=1200.
PW=S6*(PD-PV)/(S6+S7)+PV
R=(PD-PV)/(S6+S7)
F=(PW-PV)/(4.*S6)-R/2.
B=(R*S6)/3.-(PV+2.*PW)/6.
C=S6*(PW+2.*PV)/12.
SH=(-B-(B**2-4.*F*C)((.5)/(2.*A)
PM=SH*(PW-PV)/S6+PV
A(21,6)=2.+2.*SH/(S6-SH)
W(21)=(SH**2*(2.*PM+PV)+(S6-SH)**2*SH*(PW+2.*PM)/(S6-SH))/6.
F=(PD-PW)/(4.*S7)-R/2.
B=(R*S7)/3.-(PW+2.*PD)/6.
C=S7*(PD+2.*PW)/12.
SH=-B-(B**2-4.*F*C)**.5)/(2.*A)
```

```
      PM=SH*(PD-PW)/S7+PW
      A(22,7)=2.+2.*SH/(S7-SH)
      W(22)=(SH**2.*(2.*PM+PW)+(S7-SH)**2*(PD+2.*PM)*SH/(S7-SH))6.
      A(23,1)=2.+S6/S7
      A(23,4)=2.*S6/S7
      A(23,6)=2.*S6/S7
      A(23,2)=S1/S2
      A(23,7)=S6/S7
      W(23)=(S7**2*(PD-2.*PW)*S6/S7+S6**2*(2.*PW+PT)-S2**2*(2.*PX+PH)*S6
     1/S7-S1**2*(PT-2.*PX))/6.
C LOAD 2 INFLUENCE
      PA=115.
      A(24,9)=4.
      W(24)=PA*(2.*S9)**2/4.
      PC=1085.
      A(25,5)=4.
      W(25)=PC*S5**2/4.
      A(26,5)=3.
      A(26,2)=1.
      W(26)=W(25)
      A(27,2)=1.
      A(27,5)=2.
      A(27,7)=1.
      A(27,9)=1.
      W(27)=W(25)+W(24)/2.
      A(28,5)=3.
      A(28,7)=1.
      A(28,9)=1.
      W(28)=W(27)
      PH=1085.
      PX=S1*PH/(S1+S2)
      R=PH/(S1+S2)
```

```
 F=(PH-PX)/(4.*S2)-R/2.
 B=R*S2/3.-(PX+2.*PH)/6.
 C=S2*(PH+2.*PX)/12.
 SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
 A(29,2)=2.+2.*(S2-SH)/SH
 PM=SH*(PH-PX)/S2+PX
 W(29)=(SH**2*(2.*PM+PX)*(S2-SH)/SH+(S2-SH**2*(PH+2.*PM))/6.
 A(30,2)=1.+2.*(S2-SH)/SH
 A(30,5)=1.
 W(30)=W(29)
 A(31,2)=2.+SH/S5
 A(31,5)=SH/S5
 W(31)=PC*S5**2*SH/(S5*2.)+(SH**2*(2.*PM+PX)+(S2-SH**2*(PH+2.*PM)*
2SH/S5)6.
 A(32,2)=S5/(2.*S2)
 A(32,5)=2.+S5/S2
 W(32)=PC*(S5/2.)**2/2.+PC*(S5/2.)**2*S5/(4.*S2)+S5*S2*(2.*PH+PX)/
312.
 SH=S1*(1.-(1./3.)**.5)
 A(33,1)=2.+2.*(S1-SH)/SH
 PM=PX(S1-SH)/S1
 W(33)=(SH**2*(S1-SH)*(PX+2.*PM)/SH+(S1-SH)**2*2.*PH)/6.
 A(34,1)=1.+2.*(S1-SH)/SH
 A(34,3)=1.
 W(34)=W(33)
 PV=95.
 A(35,8)=4.
 W(35)=PV*(2.*S8)**2/4.
 PV=95.
 PA=1200.
 R=(PA-PV)/(S6+S7)
```

```
      PW=S6*R+PV
      F=(PW-PV)/(4.*S6)-R/2.
      B=R*S6/3.-(PV+2.*PW)/6.
      C=S6*(PW+2.*PV)/12.
      SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
      PM=SH*(PW-PV)/S6+PV
      A(36,6)=2.+2.*SH/(S6-SH)
      W(36)=(SH**2*(2.*PM+PV)+(S6-SH)**2*SH*(PW+2.*PM)/(S6-SH))/6.
      F=(PA-PW)/(4.*S7)-R/2.
      B=R*S7/3.-(PW+2.*PA)/6.
      C=S7*(PA+2.*PW)/12.
      SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
      PM=SH(PA-PW)/S7+PW
      A(37,7)=2.+2.*SH/(S7-SH)
      W(37)=(SH**2*(2.*PM+PW)+(S7-SH)**2*SH*(PA+2.*PM)/(S7-SH))/6.
C LOAD 3 INFLUENCE
      A(38,9)=4.
      PA=1015.
      W(38)=(2.*S9)**2*PA/4.
      PC=320.
      A(39,5)=4.
      W(39)=S5**2*PC/4.
      A(40,2)=1.
      A(40,5)=3.
      W(40)=W(39)
      D=625.
      PH=320.
      SH=(1.-(D/S2)**.5)
      A(41,2)=2.+2.*SH/(S2-SH)
      W(41)=D**2*PH/6.
      A(42,5)=1.
```

```
      A(42,2)=1.+2.*SH/(S2-SH)
      W(42)=W(41)
      PV=0.
      PA=1335.
      PW=PA*S6/(S6+S7)
      SH=S6*(1.-(1./3.)**.5)
      PM=PA*(S6-SH)/(S6+S7)
      A(43,6)=2.+2.*SH/(S6-SH)
      W(43)=((S6-SH)**2*PM+SH**3*(PW+2.*PM)/(S6-SH))/6.
      R=PA/(S6+S7)
      F=(PA-PW)/(4.*S7)-R/2.
      B=R*S7/3.-(PW+2.*PA)/6.
      C=S7*(PA+2.*PW)/12.
      SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
      PM=R*(S6+SH)
      A(44,7)=2.+2.*SH/(S7-SH)
      W(44)=(SH**2*(2.*PM+PW)+(S7-SH)**2*SH*(PA+2.*PM)/(S7-SH))/6.
C LOAD 4 INFLUENCE
      PA=875.
      A(45,9)=4.
      W(45)=PA*(2.*S9)**2/4.
      PD=875.
      A(46,5)=4.
      W(46)=PD*S5**2/4.
      A(47,2)=1.
      A(47,5)=3.
      W(47)=W(46)
      SH=0.53*S1
      PG=875.
      PO=50.
      PX=(PG-PO)/S1+S2-875.)+PA
      A(48,2)=2.+2.*SH/(S1-SH)
```

```
W(48)=SH**2*PG/2.+(S1-SH)*PG/2.
A(49,2)=2.+SH/(S2-SH)
A(49,5)=SH/(S2-SH)
W(49)=W(48)
R=(PG-PO)/1625.
F=(PX-PO)/(4.*S1)-R/2.
B=R*S1/3.-(PO+2.*PX)/6.
C=S1*(PX+2.*PO)/12.
SH=(-B-(B**2-4.*F*C)**.5)/(2.*A)
PM=R*SH+PO
A(50,1)=2.+2.*SH/(S1-SH)
W(50)=(SH**2*(2.*PM+PO)+SH*(S1-SH)**2*(PX+2.*PM)/(S1-SH))/6.
A(51,1)=1.+2.*SH/(S1-SH)
A(51,3)=1.
W(51)=W(50)
PO=50.
PT=0.
SH=S3*(1.-(1./3.)**.5)
A(52,3)=2.+2.*SH/(S3-SH)
PM=PO*(S3-SH)/S3
W(52)=(SH**2*(PO+2.*PM)+(S3-SH)*2.*PM)/6.
PD=1085.
A(53,3)=2.
A(53,4)=2.
A(53,5)=2.
W(53)=S5**2*PD/2.
PUNCH 12,S2,S5
DO 6 J=1,9
DO 5 I=10,53
ANMBR=A(I,J)
IF (ANMBR .EQ.O.) GO TO 5
PUNCH 10, J, I, A(I,J)
```

```
 5    CONTINUE
 6    CONTINUE
      DO 7 I=10,53
      PUNCH 11, I, W(I)
 7    CONTINUE
 10   FORMAT (1H ,3X,'MOM',I1,6X,'MECH',I2,4X,F9.7)
 11   FORMAT (1H ,3X,'WORK',6X,'MECH',I2,4X,E12.3)
 12   FORMAT (1H ,4X,'S2=',F6.1,'S5=',F6.1)
      GOTO 1
    2 STOP
      END
$ENTRY
  1200.     1200.

   ----------------------------

  0.0         1.0
/*
/*EOJ ********
```

The output data from the program is not listed in this document as computed. To list this information would require a separate volume.

To complete the M.P.S.X. linear programming data set, objective function cards listing the various beam lengths are added to the FORTRAN program output cards. Finally the desired M.P.S.X. control language is inserted and the problem run to get optimum upper bound values for the plastic limit moment for each beam in each frame. It should again be noted that the very rigid M.P.S.X. input format requirements outlined in the numerical routine appendix must be followed precisely. Failure to do so may result in the linear programming routine being abandoned or worse, in erroneous results.

In order to compare the web frame sizes required by the plastic limit design problem to Lund's elastic design results, the values of plastic limit moment are used to size beam elements based on the beam proportions used by Lund in his analysis. A 0.2 cm corrosion allowance is then applied to plating thicknesses. Lund based his beam proportions on a set of starting values which he scaled up to meet stress control requirements. The values of plate flange area A_p are likely to be drived by design factors not considered here and are assumed to be held constant. Lund does not list thickness

values for plate flanges. It is therefore assumed that the full area of A_p may be considered to be part of the beam to be sized. The starting values of web depth, web thickness, top flange area and plate flange area as used by Lund and in this study for scaling beams is listed in Table A.1.1.

Lund assumed his designs to be of mild steel. Stress values were restricted to 1500 kp/cm^2 versus the yield stress value of 2249.8 kp/cm^2. This gives an effective factor of safety of 1.50. There is some question as to the "proper" factor of safety to be applied to the values of plastic limit moment to make a reasonable comparison between the elastic and plastic designs.

Values of beam geometry for the plastic limit designs were calculated using factors of safety of 1.5, 2.0, and 2.5. The dimensions calculated included those listed in the heading of Table A.1.1, as well as the total cross sectional area of the beam excluding plate flange area. This total cross sectional area will henceforth be called A_T.

In calculating the values named above the following relationships were satisfied.

$$M_0 = \sigma_Y \, z_p$$

where

$$z_p = \frac{A\ a}{2}$$

In these relationships M_0 is the plastic limit moment calculated in the limit design problem, σ_y is the material yield stress, A is the total required area of the cross section and a is the distance between centroids of the two equal areas lying above and below the plastic neutral axis.

The values A_T, web depth h, web thickness t and top flange area A_T, are listed in the tables at the end of this appendix, starting with Table A.1.2. The numbers in the frame column represent values of S2 and S5 respectively. For example "12 - 14" denotes that this is the frame where S2 = 12.0 meters and S5 = 14.0 meters. Dimensions with factors of safety of 1.5 and 2.0 are listed for all beam elements exclusive of Element #4. It was found that the limiting mechanisms of the limit design problem never included this element. Those mechanisms in which failure of the member might have occurred had high values of internal energy dissipation with low external work rates when compared with the other possible collapse mechanisms.

It is known that Member #4 is in compression and subject to buckling. Plastic buckling is not considered in this study. In order to provide some measure of comparison, the dimensions found by Lund in his study are used unaltered in this analysis when final frame weights are computed. In these computations

Element #4 is assumed to have a cross sectional area of 432 cm^2 with a web height of 153 cm for all cases.

Figure A.4 illustrates variations of net frame area A_T for various factors of safety as calculated for Member #1.

Total web frame weights were calculated for one half of the complete frames. To be consistent with Lund, comparisons are based on the assumption that the relative amount of local stiffening and relative bracket sizes are equal for all investigated frames and these components are neglected. Overlaps are compensated for by subtracting one half of the average weight in the joint region for each beam. Total web frame weights were therefore calculated using the following relation:

$$W = \rho \sum_{i=1}^{9} \{l_i A_T - \tfrac{1}{2} (c_j + c_k) A_T\}$$

where ρ is material density, l_i is the member length and c_j and c_k are the overlaps at the respective beam ends. This relationship was also used by Lund [5].

The weight in metric tons of each frame considered is listed in Table A.1.18. Weights were calculated based on factors of safety of 1.5 and 2.0.

Lund assumed a constant cross-tie height of 12.0 meters. The limit design frame group with the 12.0 meter cross-tie height had the lowest weights, as a group. Figure A.5 shows values of frame weight

versus side tank breadth for single cross-tie frames with cross-tie heights of 12.0 meters. Solid lines represent limit design results. Dashed lines represent all values for single cross-tie frames given by Lund. There are two curves joining the elastic frame weights. The one to the upper right represents weight values where the bulkhead girders are outboard of the longitudinal bulkhead. The remaining elastic design curve gives weight values for frames with bulkhead girders inboard of the longitudinal bulkhead. The two limit design curves both assume the bulkhead girders to to inboard of the longitudinal bulkhead. The two curves give weights for factors of safety of 1.5 and 2.0 for the various frames.

TABLE A.1.1

Initial Member Dimensions on which Beam Proportions are Based

Member No.	Height (cm)	Web Thickness (cm)	Top Flange Area (cm^2)	Plate Flange Area (cm^2)
1	300	1.5	120	660
2	300	1.5	120	660
3	300	1.4	80	750
4	160	2.0	150	150
5	400	1.8	120	750
6	300	1.5	120	420
7	300	1.5	120	510
8	240	1.25	80	750
9	400	2.0	120	750

Table A.1.2 - Dimensions of Member #1
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	428.93	244.63	1.42	79.79
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	388.41	232.04	1.36	71.79
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	348.82	219.09	1.30	64.00
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	311.22	206.09	1.33	56.63
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	309.06	205.32	1.33	56.21
14-14	305.83	204.17	1.32	55.58
14-16	275.61	193.04	1.17	49.68
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.3 - Dimensions of Member #2
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	613.10	295.49	1.68	116.42
10-14	642.58	302.88	1.71	122.32
10-16	669.92	309.58	1.75	127.79
10-18	699.33	316.64	1.78	133.68
10-20	728.71	323.55	1.82	139.58
11-12	658.35	306.77	1.73	125.47
11-14	687.78	313.89	1.77	131.37
11-16	718.22	321.10	1.81	137.47
11-18	748.63	328.15	1.84	143.58
11-20	782.15	335.76	1.88	150.32
12-12	701.43	317.14	1.79	134.11
12-14	733.95	324.77	1.82	140.63
12-16	766.44	332.22	1.86	147.16
12-18	799.95	339.74	1.90	153.89
12-20	833.42	347.09	1.94	160.63
13-12	745.49	327.43	1.84	142.95
13-14	780.06	335.29	1.88	149.89
13-16	813.55	342.74	1.91	156.63
13-18	848.05	350.26	1.95	163.58
13-20	884.61	358.06	1.99	170.95
14-12	771.68	333.40	1.87	148.21
14-14	811.46	342.28	1.91	156.21
14-16	859.55	352.73	1.96	165.89
14-18	897.14	360.70	2.00	173.47
14-20	932.61	368.07	2.04	180.63

Table A.1.4 - Dimensions of Member #3
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	426.89	259.46	1.41	59.84
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	386.19	245.93	1.35	53.76
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	346.40	231.99	1.28	47.84
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	309.68	218.40	1.22	42.40
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	113.03	125.14	.78	13.92
14-14	135.76	138.78	.85	17.12
14-16	273.87	204.35	2.15	37.12
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.5 - Dimensions of Member #5
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	956.60	408.01	2.04	124.86
10-14	1216.50	462.50	2.28	160.43
10-16	1488.72	513.62	2.51	197.86
10-18	1766.22	561.12	2.73	236.14
10-20	2048.14	605.69	2.93	275.14
11-12	980.67	413.35	2.06	128.14
11-14	1237.32	466.60	2.30	163.29
11-16	1508.43	517.13	2.53	200.57
11-18	1783.80	564.00	2.74	238.57
11-20	2064.64	608.20	2.94	277.43
12-12	1003.68	418.39	2.08	131.29
12-14	1257.09	470.46	2.32	166.00
12-16	1526.05	520.26	2.54	203.00
12-18	1800.34	566.69	2.75	240.86
12-20	2079.08	610.39	2.95	279.43
13-12	1022.49	422.46	2.10	133.86
13-14	1274.78	473.89	2.33	168.43
13-16	1541.60	523.00	2.55	205.14
13-18	1813.78	568.88	2.76	242.71
13-20	2092.48	612.41	2.96	281.29
14-12	1099.77	438.83	2.17	144.43
14-14	1338.20	485.99	2.39	177.14
14-16	1556.11	525.54	2.56	207.14
14-18	1827.21	571.05	2.77	244.57
14-20	2104.85	614.27	2.96	283.00

Table A.1.6 - Dimensions of Member #6
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	384.13	230.67	1.35	70.95
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	342.38	216.92	1.28	62.74
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	304.76	203.78	1.22	55.37
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	269.11	190.57	1.15	48.42
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	234.36	176.81	1.08	41.68
14-14	"	"	"	"
14-16	"	"	"	"
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.7 - Dimensions of Member #7
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	372.38	226.88	1.33	68.63
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	414.01	240.07	1.40	76.84
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	454.45	252.25	1.46	84.84
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	494.78	263.88	1.52	92.84
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	535.00	275.01	1.58	100.84
14-14	"	"	"	"
14-16	"	"	"	"
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.8 - Dimensions of Member #8
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	272.67	188.33	1.18	49.26
10-14	221.67	168.36	1.08	39.37
10-16	173.45	147.23	.97	30.11
10-18	127.87	124.34	.85	21.47
10-20	85.85	99.26	.72	13.68
11-12	272.67	188.33	1.18	49.26
11-14	221.67	168.36	1.08	39.37
11-16	173.45	147.23	.97	30.11
11-18	127.87	124.34	.85	21.47
11-20	85.85	99.26	.72	13.68
12-12	272.67	188.33	1.18	49.26
12-14	221.67	168.36	1.08	39.37
12-16	173.45	147.23	.97	30.11
12-18	127.87	124.34	.85	21.47
12-20	85.85	99.26	.72	13.68
13-12	272.67	188.33	1.18	49.26
13-14	221.67	168.36	1.08	39.37
13-16	173.45	147.23	.97	30.11
13-18	127.87	124.34	.85	21.47
13-20	85.85	99.26	.72	13.68
14-12	272.67	188.33	1.18	49.26
14-14	221.67	168.36	1.08	39.37
14-16	173.45	147.23	.97	30.11
14-18	127.87	124.34	.85	21.47
14-20	85.85	99.26	.72	13.68

Table A.1.9 - Dimensions of Member #9
(Factor Safety = 1.5)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	1278.87	454.35	2.47	154.83
10-14	1031.25	406.26	2.23	123.78
10-16	800.16	355.82	1.98	94.96
10-18	581.14	300.72	1.70	67.83
10-20	381.06	240.29	1.40	43.30
11-12	1278.87	454.35	2.47	154.83
11-14	1031.25	406.26	2.23	123.78
11-16	800.16	355.82	1.98	94.96
11-18	581.14	300.72	1.70	67.83
11-20	381.06	240.29	1.40	43.30
12-12	1278.87	454.35	2.47	154.83
12-14	1031.25	406.26	2.23	123.78
12-16	800.16	355.82	1.98	94.96
12-18	581.14	300.72	1.70	67.83
12-20	381.06	240.29	1.40	43.30
13-12	1278.87	454.35	2.47	154.83
13-14	1031.25	406.26	2.23	123.78
13-16	800.16	355.82	1.98	94.96
13-18	581.14	300.72	1.70	67.83
13-20	381.06	240.29	1.40	43.30
14-12	1278.87	454.35	2.47	154.83
14-14	1031.25	406.26	2.23	123.78
14-16	800.16	355.82	1.98	94.96
14-18	581.14	300.72	1.70	67.83
14-20	381.06	240.29	1.40	43.30

Table A.1.10 - Dimensions of Member #1
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	514.90	269.50	1.55	96.84
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	465.07	255.36	1.48	86.95
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	417.21	241.05	1.41	77.47
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	372.38	226.88	1.33	68.63
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	370.24	226.18	1.33	68.21
14-14	364.89	224.43	1.32	67.16
14-16	329.50	212.50	1.26	60.21
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.11 - Dimensions of Member #2
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	736.05	325.25	1.83	141.05
10-14	770.63	333.17	1.87	148.00
10-16	804.13	340.66	1.90	154.74
10-18	840.74	348.68	1.94	162.11
10-20	876.26	356.30	1.98	169.26
11-12	790.53	337.64	1.89	152.00
11-14	826.10	345.50	1.93	159.16
11-16	862.68	353.40	1.97	166.53
11-18	900.27	361.36	2.01	174.11
11-20	939.91	369.57	2.05	182.11
12-12	842.83	349.13	1.95	162.53
12-14	881.48	357.40	1.99	170.32
12-16	921.14	365.70	2.03	178.32
12-18	960.76	373.81	2.07	186.32
12-20	1001.39	381.96	2.11	194.53
13-12	895.05	360.26	2.00	173.05
13-14	936.78	368.92	2.04	181.47
13-16	977.44	377.18	2.09	189.68
13-18	1020.13	385.66	2.13	198.32
13-20	1038.87	389.33	2.15	202.11
14-12	927.40	366.99	2.03	179.58
14-14	975.35	376.76	2.08	189.26
14-16	1033.66	388.32	2.14	201.05
14-18	1078.39	396.96	2.18	210.11
14-20	1122.05	405.23	2.23	218.95

Table A.1.12 - Dimensions of Member #3
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	511.11	285.55	1.53	72.48
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	462.13	270.67	1.46	65.12
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	415.12	255.62	1.39	58.08
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	370.07	240.37	1.32	51.36
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	134.63	138.13	.84	16.96
14-14	161.59	152.97	.91	20.80
14-16	326.98	224.90	1.25	44.96
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.13 - Dimensions of Member #5
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	1149.83	449.13	2.22	151.29
10-14	1462.79	508.97	2.49	194.29
10-16	1792.07	565.35	2.74	239.71
10-18	2126.50	617.52	2.98	286.00
10-20	2467.32	666.62	3.20	333.29
11-12	1133.80	444.00	2.11	134.86
11-14	1488.72	513.62	2.51	197.86
11-16	1815.84	569.21	2.76	243.00
11-18	2148.15	620.75	2.99	289.00
11-20	2486.87	669.33	3.21	336.00
12-12	1206.09	460.43	2.27	159.00
12-14	1512.57	517.87	2.53	201.14
12-16	1837.54	572.71	2.78	246.00
12-18	2167.73	623.66	3.01	291.71
12-20	2505.38	671.88	3.22	338.57
13-12	1230.03	465.17	2.29	162.29
13-14	1533.31	521.54	2.55	204.00
13-16	1856.14	575.70	2.79	248.57
13-18	2185.25	626.25	3.02	294.14
13-20	2520.80	674.01	3.23	340.71
14-12	1322.61	483.05	2.37	175.00
14-14	1609.98	534.88	2.61	214.57
14-16	1872.67	578.34	2.80	250.86
14-18	2200.71	628.53	3.03	296.29
14-20	2535.20	675.98	3.24	342.71

Table A.1.14 - Dimensions of Member #6
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	459.76	253.81	1.47	85.89
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	410.82	239.08	1.40	76.21
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	364.89	224.43	1.32	67.16
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	320.90	209.51	1.25	58.53
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	279.93	194.67	1.17	50.53
14-14	"	"	"	"
14-16	"	"	"	"
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.15 - Dimensions of Member #7
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	445.95	249.74	1.45	83.16
10-14	"	"	"	"
10-16	"	"	"	"
10-18	"	"	"	"
10-20	"	"	"	"
11-12	495.84	264.18	1.52	93.05
11-14	"	"	"	"
11-16	"	"	"	"
11-18	"	"	"	"
11-20	"	"	"	"
12-12	545.57	277.87	1.59	102.95
12-14	"	"	"	"
12-16	"	"	"	"
12-18	"	"	"	"
12-20	"	"	"	"
13-12	594.13	290.64	1.65	112.63
13-14	"	"	"	"
13-16	"	"	"	"
13-18	"	"	"	"
13-20	"	"	"	"
14-12	642.58	302.88	1.71	122.32
14-14	"	"	"	"
14-16	"	"	"	"
14-18	"	"	"	"
14-20	"	"	"	"

Table A.1.16 - Dimensions of Member #8
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	326.50	207.48	1.28	59.79
10-14	265.10	185.50	1.17	47.79
10-16	206.39	161.94	1.04	36.42
10-18	152.42	137.10	.91	26.11
10-20	101.89	109.43	.77	16.63
11-12	326.50	207.48	1.28	59.79
11-14	265.10	185.50	1.17	47.79
11-16	206.39	161.94	1.04	36.42
11-18	152.42	137.10	.91	26.11
11-20	101.89	109.43	.77	16.63
12-12	326.50	207.48	1.28	59.79
12-14	265.10	185.50	1.17	47.79
12-16	206.39	161.94	1.04	36.42
12-18	152.42	137.10	.91	26.11
12-20	101.89	109.43	.77	16.63
13-12	326.50	207.48	1.28	59.79
13-14	265.10	185.50	1.17	47.79
13-16	206.39	161.94	1.04	36.42
13-18	152.42	137.10	.91	26.11
13-20	101.89	109.43	.77	16.63
14-12	326.50	207.48	1.28	59.79
14-14	265.10	185.50	1.17	47.79
14-16	206.39	161.94	1.04	36.42
14-18	152.42	137.10	.91	26.11
14-20	101.89	109.43	.77	16.63

Table A.1.17 - Dimensions of Member #9
(Factor Safety = 2.0)

Frame (S2 - S5)	A_T (cm^2)	h (cm)	t (cm)	A_F (cm^2)
10-12	1539.02	500.09	2.70	187.57
10-14	1240.44	447.21	2.44	150.00
10-16	960.29	391.43	2.16	114.91
10-18	698.25	331.27	1.86	82.30
10-20	455.88	264.41	1.52	52.43
11-12	1539.02	500.09	2.70	187.57
11-14	1240.44	447.21	2.44	150.00
11-16	960.29	391.43	2.16	114.91
11-18	698.25	331.27	1.86	82.30
11-20	455.88	264.41	1.52	52.43
12-12	1539.02	500.09	2.70	187.57
12-14	1240.44	447.21	2.44	150.00
12-16	960.29	391.43	2.16	114.91
12-18	698.25	331.27	1.86	82.30
12-20	455.88	264.41	1.52	52.43
13-12	1539.02	500.09	2.70	187.57
13-14	1240.44	447.21	2.44	150.00
13-16	960.29	391.43	2.16	114.91
13-18	698.25	331.27	1.86	82.30
13-20	455.88	264.41	1.52	52.43
14-12	1539.02	500.09	2.70	187.57
14-14	1240.44	447.21	2.44	150.00
14-16	960.29	391.43	2.16	114.91
14-18	698.25	331.27	1.86	82.30
14-20	455.88	264.41	1.52	52.43

Table A.1.18

Limit Design Frame Weights for Single Cross-Tie Tanker

Frame (S2 - S5)	Frame Weight (metric tons) FS = 1.5	Frame Weight (metric tons) FS = 2.0
10-12	41.77	48.89
10-14	42.71	49.09
10-16	44.82	51.77
10-18	48.96	56.44
10-20	56.08	63.45
11-12	41.55	47.85
11-14	41.99	48.63
11-16	44.41	51.13
11-18	48.67	55.94
11-20	54.77	62.77
12-12	41.47	48.41
12-14	41.90	48.50
12-16	44.29	50.96
12-18	48.53	55.59
12-20	54.57	62.48
13-12	41.72	48.65
13-14	42.14	48.70
13-16	44.50	51.09
13-18	48.68	55.71
13-20	54.73	62.34
14-12	41.67	48.51
14-14	42.89	48.40
14-16	44.92	51.54
14-18	49.11	56.13
14-20	55.09	62.85

APPENDIX B - TANKER FRAME WITH RIGID CORNERS

In large ships which have relatively simple web frame geometries, the webs of the individual beams are likely to be quite deep. Brackets are also rather large.

In this study, one such frame is examined using the assumption of rigid extensions in the bracket and beam overlap regions. The web frame numbered OC22,49.2t was selected from Lund's paper [5] as a representative frame from which a model could be developed. This frame was among the lighter frames having zero cross-ties and is shown in Figure 10 of Reference [5].

The frame model is shown in Figure A.3. The dots on the system lines represent the end points of the rigid extensions. Here, as in Appendix A., the system lines are assumed to be defined on the inside surface of the shell plating and on the center tank face of the longitudinal bulkhead plating. Plastic neutral axes of the individual beam elements calculated did in fact lie on or very near these lines. The principal dimensions of the ship under consideration are the same as those listed in Appendix A.

The loading conditions considered are those defined by Lund [5], discussed in Appendix A and shown

in Figure A.2.

The assumed lengths of bracket legs are identical to those used by Lund and are listed in Table 2 of Reference [5].

The lengths of the rigid extensions were calculated by hand using the procedures outlined in Section 2.2 and Reference [6]. Although Lund only calculates these values once for each frame, he does not list his results. It is therefore assumed, but not known as fact that rigid extension lengths used in this study are approximately equal to values used by Lund for the same frame. The actual values used are listed in Table A.2.1.

In this analysis the calculations required to develop the limit design problem were done by hand. The assumption of rigid joint regions required that each proposed collapse mechanism be carefully inspected in insure kinematic admissibility. Joint rotations were not considered as fundamental mechanisms in the formations of mechanisms which involved more than one frame element. A discussion of the joint rotation problem is given in Section 2.3.

The results of the hand calculations are listed in Table A.2.2 The values of the constraint matrix coefficients (i.e. internal energy dissipation coefficients) as well as the values of the constraint

equation right hand sides (i.e. external work rates) are tabulated.

It should be noted that although the values of constraint relations are shown in the table to be segregated by load condition, they were combined as one system in the linear programming problem. This allowed the system to be truly optimized for all considered loadings. In general, this is not possible in the elastic design methods. Most elastic procedures optimize for each load condition and then select the larger dimensions required for each element.

The linear programming subroutine M.P.S.X. is used to optimize the results of the hand calculations as discussed in Appendix C. Again, as in Appendix A, the output from the procedure is a listing of minimum values of plastic limit moment required to make the structure safe under the prescribed loading conditions.

The dimensions of the beam elements were calculated based on the relation

$$M_0 = \sigma_y z_p$$

where

$$z_p = \frac{A\ a}{2}$$

In these relationships M_0 is the plastic limit moment calculated in the limit design problem, σ_y is the yield stress of the material, A is the total required area of the cross section and a is the distance between

centroids of the two equal areas lying above and below the plastic neutral axis.

In order to allow comparison to the parent Lund frame, the beam elements are sized to meet the plastic moment requirements based upon the proportions of the beams which Lund used as starting values for his frame iterations. For this frame, starting values of web depth, web thickness and top flange area are shown in Table A.2.3. Also shown are values of plate area which are assumed to remain constant.

Values of beam geometry were calculated using factors of safety of 1.0, 1.5, 2.0 and 2.5. A corrosion allowance of 0.2 cm was applied. Table A.2.4 gives resulting values of net beam cross sectional area A_T, web depth h, web thickness t and top flange area A_F.

Frame weights were calculated for one half of complete frames for each of the factors of safety considered. The relation

$$W = \rho \sum_{i=1}^{6} \{l_i A_T - \tfrac{1}{2} (c_j + c_k) A_T\}$$

was used to calculate these weight values where ρ is material density, l_i is the member length and c_j and c_k are the overlaps at the respective beam ends. This expression was also used by Lund. Consistent with Lund's calculations, weight of web stiffeners and brackets are neglected.

Values of frame weight are plotted against factors of safety for this analysis and are shown in Figure A.6. Also plotted is the weight of the Lund elastic design frame at a factor of safety of 1.5.

Table A.2.1 - Rigid Extension Lengths

Rigid Extension	Length (cm)	Rigid Extension	Length (cm)
L_B	504	L_M	453
L_C	630	L_N	453
L_D	128	L_O	153
L_E	428	L_P	392
L_F	553	L_Q	382

Table A.2.2 - Coefficients of Limit Design Equations

Load #1 Mechanisms -

	M_1	M_2	M_3	M_4	M_5	M_6	RHS	EXP
A	4.0						3.8541	8
B	1.0	1.3033		1.7935			7.9294	7
C		4.0					2.0480	7
D		3.0	.7546				5.9997	7
E	1.4228	1.7847	.7847	1.7974			2.6842	8
F	1.8456	3.0		1.7934			1.7293	8
G			4.1112				7.3395	7
H			2.9060		.9060		1.3003	8
I		.8996	2.8996				1.3389	8
J					4.0776		1.7761	7
K			.8478		2.8478		5.5439	7
L				3.6632			3.4642	8
M	1.8456	1.3034			2.6224		3.2292	8
N				3.5920	1.3834	1.5536	5.7481	8
O		2.0			2.1152		1.5717	6

Table A.2.2 - Continued

Load #2 Mechanisms -

	M_1	M_2	M_3	M_4	M_5	M_6	RHS	EXP
A	4.0						3.4270	4
B	1.0	1.3034		1.7974			4.2134	8
C		4.0					1.9322	8
D		2.9193	.9193				1.8339	9
E		2.2967	.2967				7.7492	8
F		3.0	1.8456	1.7974			5.0677	8
G			3.6346				2.6684	8
H			2.6307		.6307		3.8264	8
I		1.2508	3.2508				8.8934	8
J						4.0	4.7620	7
K				1.4812	1.3834	1.0	1.0366	8
L				3.6632			3.4649	8
M				3.5920	1.3924	1.5536	5.0815	8
N	1.8456	1.3034		2.6214			7.9528	8
O		2.0			2.1152		8.7776	8

Table A.2.2 - Continued

Load #3 Mechanisms -

	M_1	M_2	M_3	M_4	M_5	M_6	RHS	EXP
A	4.0						3.6053	8
B	1.0	1.3034		1.7934			4.2284	7
C		4.0					2.0731	7
D	1.8456	3.0		1.7934			1.0665	8
E		2.8558	.8558				2.8267	8
F			2.0884				4.0853	4
G			2.0640				3.1770	4
H		2.1371	4.1370				1.0762	8
I				4.4800			4.3311	8
J	12.1690	8.5938		16.9852			1.4932	9
K				2.7326	1.0136	1.1378	4.3594	8
L		2.0			2.1160		2.3637	8
M		1.0	2.0428	1.0428			2.5891	8
N			2.0203	4.0406		1.0203	9.4776	8
O	1.0	1.0		4.0406	1.0210	1.0210	9.8255	8
P	1.0	2.0	1.2658	1.7934			1.0523	8

Table A.2.2 - Continued

Load #4 Mechanisms -

	M_1	M_2	M_3	M_4	M_5	M_6	RHS	EXP
A	4.0						3.1081	8
B	1.0	1.3034		1.7934			2.1056	8
C		4.0					1.5582	8
D			3.6704				3.3968	8
E					4.5213		3.5962	6

Table A.2.3 - Initial Member Dimensions

Member No.	Height (cm)	Web Thickness (cm)	Top Flange Area (cm^2)	Plate Flange Area (cm^2)
1	400	2.0	120	750
2	400	1.8	120	750
3	300	1.5	120	660
4	300	1.5	120	420
5	300	1.4	80	750
6	240	1.25	80	750

Table A.2.4 - Dimensions of Limit Design Results

	A_T	h	t	A_F
FS = 1.0				
1	321.00	236.52	1.28	42.07
2	1130.00	463.94	2.29	161.43
3	415.00	255.64	1.48	87.35
4	533.00	264.56	1.52	93.03
5	40.00	84.77	.60	6.39
6	98.00	96.01	1.09	12.65
FS = 1.5				
1	475.02	270.27	1.55	54.78
2	1445.16	505.78	2.48	191.86
3	601.51	292.54	1.66	114.11
4	640.47	302.36	1.71	121.89
5	71.16	95.81	.65	8.16
6	123.38	121.88	.83	20.63
FS = 2.0				
1	596.51	296.53	1.69	66.39
2	1739.34	556.69	2.71	232.43
3	722.95	322.21	1.81	138.42
4	769.03	332.80	1.86	147.68
5	84.32	105.78	.69	9.95
6	153.95	137.86	.92	26.39
FS = 2.5				
1	655.07	320.00	1.80	76.96
2	2009.29	599.74	2.90	269.76
3	833.37	347.08	1.94	160.62
4	887.19	358.61	1.99	171.47
5	96.14	114.09	.73	11.57
6	176.69	148.74	.97	30.73

APPENDIX C - NUMERICAL ROUTINE

C.1 Use of the M.P.S.X.

The M.P.S.X. linear programming subroutine used in this analysis was found to be very sensitive to format errors. The user's guides and instructions available on the system are not very careful to point out possible pit falls. It is the purpose of this section to list the control language statements used in solving the linear programming problems of this analysis and to point out general problems that were encountered in using the M.P.S.X.. Before attempting to use this subroutine the user should become very familiar with References [31 - 33].

A sample listing of the control program with sample data points follows:

```
//LP  EXEC MPSX
//MPSCOMP.SYSIN  DD  *
         PROGRAM
         INITIALZ
         MOVE(XDATA,'LUND')
         MOVE(XPBNAME,'PBFILE')
         CONVERT('SUMMARY')
         BCDOUT
         SETUP
         MOVE(XOBJ,'MINWT')
         MOVE(XRHS,'WORK')
         CRASH
         PRIMAL
         SOLUTION
         RANGE
         EXIT
         PEND
/*
//MPSEXEC.SYSIN  DD  *
```

```
NAME            LUND
ROWS
 G  MECHA
---------
COLUMNS
    MOM1        MECHA     4.0              MECHB     1.0
---------
RHS
    WORK        MECHA     3.605E+8         MECHB     4.228E+7
---------
ENDATA
/*
```

Explanations of the control language statements can be found in Reference [33]. The format sensitivity of the M.P.S.X. has been mentioned previously. The greatest difficulty occurs when entering data. Page 209 of Reference [33] will be found to be invaluable to any prospective system user. This page tabulates all required data entries in the positions required by the M.P.S.X.. The data set name and all data vector names must be left justified in their fields. Failure to have the data set name left justified will result in having the data set not read. If a row or vector name is not left justified, its associated data point will not be read. If a row or vector name extends beyond the left bound of its field, the data will be read and labeled with an erroneous name. Any of these errors associated with row and vector names will result in erroneous solutions.

The scale values of the data may be placed anywhere in the prescribed field.

Finally, Chapter 4 of Reference [33] should be checked to insure that data and vector names use only "legal" characters. The use of blanks and certain other characters will result in abandoned runs.

C.2 M.P.S.X. Costs

The two positive advantages of the M.P.S.X. are its power and efficiency. The linear programming problems solved in this analysis are moderately large, having roughly fifty constraint relations and nine variables. There was originally some fear that the solutions of these problems would be quite expensive.

It was found that problems having approximately 50 constraints and 6 variables required approximately 0.325 minutes in total job run time at a cost of approximately four dollars. These numbers are based on computation performed in solving the linear programming problem of Appendix B.

Run time and associated costs were found to be very closely related to the total number of constraints. Changes in the number of solution variables had little or no effect on computation time.

The total cost of running the limit design program in Appendix A for twenty-five different frames including the card punching cost was $17.57. The cost of solving the programming problems formed by the FORTRAN program was $74.08. This resulted in a total computation

cost of \$91.65 for limit design problem solutions of twenty-five frame configurations.

Thesis 171170
K455 Kinghorn
Limit design of ship
transverse web frames.